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INSTITUTULUI POLITEHNIC „TRAIAN VUIA"
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1983.

LUCRĂRILE SEMINARULUI DE MATEMATICĂ ŞI FIZICĂ

Noiembrie 1983

INSTITUTUL POLITEHNIC TRAIAN VUIA DIN TIMISOARA MCMXX

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lucrarile seminarului de MATEMATICA si FIZICA

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NOIEMBRIE 1983

MŰSZAKI EGYETEM KÖNYVTÁRA 1983

INTREPRINDEREA DE CONSTRUCTII DE MASINI
R E S I T A - R O M A N I A
PRODUCE SI OFERA

- HIDROAGREGATE DESTINATE ECHIPARII CENTRALELOR HIDROELECTRICE

	Caderi m	Puteri Kw
KAPLAN	4,85 - 38	55o - 178.ooo
FRANCIS	2o - 526	747 - 17o.ooo
BULB	7,45 - 12,8	216 - 28.ooo

- MICROHIDROAGREGATE DE TIP EOS SI FO PENTRU AMENAJARI HIDROELECTRICE CU PUTERI PINA LA 12oo KW
- ECHIPAMENT HIDROMECANIC
- Servomotoare cu cursă pînă la 2o m şi diametre pînă la 6oo mm
- Vane fluture cu diametrul pînă la 4ooo mm,
- Vane sferice cu diametrul pînă la 22oo mm.
- MOTOARE DIESEL NAVALE SEMIRAPIDE SI LENTE CU PUTERI DE LA 3OOO CP PINA LA 3O.OOO CP MOTOARE DIESEL RAPIDE CU PUTERI DE LA 1.5OO CP LA 4.OOO CP FOLOSITE LA:
- Tracţiune feroviară, propulsie navală, grupuri diesel generatoare pentru intervenţie, pentru foraj , auxiliare navale,etc.
- MASINI ELECTRICE SPECIALE:
- Motoare de curent continuu de la 41o la 54oo Kw
- Motoare asincrone trifazate de la 5oo la 72oo Kw
- Motoare sincrone trifazate de la 3oo la 1o.5oo KVA
- Generatoare sincrone trifazate de la 1ooo la 4ooo KVA
- ELECTROCOMPRESOARE DE AER CU DEBIT DE LA 15 LA 1OO m^3/mi
- Utilaj greu pentru industria metalurgică
- Laminoare de ţevi - tablă- profile, etc.
- Aparate de încărcare furnal
- Exhaustoare şi ventilatoare cu debite peste 5ooooo m^3/h
- UTILAJ CHIMIC DIVERS
- Coloane distilare
- Evaporatoare

LUCRARILE SEMINARULUI DE MATEMATICA SI FIZICA
AL INSTITUTULUI POLITEHNIC "TRAIAN VUIA"TIMISOARA
NOIEMBRIE 1983

SUMAR

1. DORINA RENDI , Vectori tangenţi speciali şi aplicaţii diferenţiale.

2. B.RENDI , Secţiuni admise în fibrate definite de algebre Banach comutative.

3. P.STAVRE, F.C.KLEPP , Hipersuprafeţe D-Weyl izoterme şi invarianţi scalari asociaţi.

4. O.LIPOVAN , σ- algebre cu σ - submăsuri probabiliste.

5. M.Gh.TUDOR , Studiu asupra distribuţiei zerourilor funcţiilor analitice.

6. N.COFAN, I.STAN, N.ZOPOTA , Asupra echivalenţei metodelor reale de interpolare K şi T cu metodele de medie.

7. IOSEFINA MIHUT , Clase Ehern de varietăţi foliate.

8. D.DAIANU , Morfisme speciale în categorii fuzzy şi scufundări (3) .

9. N.CRIVAT , T.BĂNZARU , Asupra quasi continuităţii limitelor şirurilor generalizate de multifuncţii.

10. DOBRINCA MIHAILOV , Asupra unor ideale de operatori pe cupluri Banach.

11. St.MATEI , Funcţii pe structuri numeroidale.

12. Gh.BABESCU , Proprietăţi de regularitate şi uniform continuitate pentru clasele de operatori cosinus şi sinus.

13. I.GOLET , Asupra unor ecuaţii în $\widetilde{\mathcal{T}}_T^S$- semigrupuri.

14. E.BURLACU, B.CRSTICI, M.NEAGU , Asupra omomorfismelor grupului activ $\mathbb{R}$ în unele grupuri de relaţii binare liniare reale.

15. P.NASLAU , Regularizarea perturbaţiilor singulare ale unor ecuaţii diferenţiale liniare în cazuri critice.

16. N.NEUHAUS , Asupra unui hipergrupoid.

17. N.N.NEAMTU , Transformarea Laplace-Dirichlet bilaterală.

18. P.GAVRUTA, T.CEAUSU , Asupra operatorului de multiplicare pe spaţii de funcţii.

19. F.RADU , Spaţiul liniar tangent generalizat de ordinul doi

20. ELENA TOPUZU , Observaţii asupra spectrului caracteristic al unui operator diagonal.

21. CARMENCITA UROSU , Asupra proprietăţilor laticiale ale congruenţilor.

22. EMILIA PETRISOR ,Asupra măsurabilităţii geodezicelor aleatoare .

23. Gh.HALIC ,Utilizarea calculului cu neutrici pentru includerea unor clase de funcţii care nu sînt local integrabile în spaţiul distribuţiilor.

24. COLETA DE SABATA,P.CRACIUN,C.MARCU, Aspecte privind efectul helio-termal. Aplicaţii în stocajul energiei solare.

25. COLETA DE SABATA, C.MARCU, I.MIRON, I.LUMINOSU , Absorbţia radiaţiei electromagnetice într-un fluid cu Fe_3O_4.

26. COLETA DE SABATA, I.MIHALCA, C.MARCU, I.DAMIAN, A.ERCUTA , Proprietăţi optice ale cupolei de sticlă utilizată în construcţia unui piranometru.

27. COLETA DE SABATA, C.MARCU, I.LUMINOSU , Asupra debitului energetic la un heliocaptator plan furnizor de aer cald tehnologic.

28. COLETA DE SABATA, C.MARCU, I.DAMIAN, T.PALADE, N.FARBAS , Dependenţa de condiţiile de obţinere a structurii păturilor subţiri electrolitice de Fe-Zn.

29. COLETA DE SABATA, C.MARCU, DOROTHEA MIHAILOVICI, I.DAMIAN, MARIA BOLDAN, MARIANA GANGAL, Asupra obţinerii şi proprietăţilor magnetice ale aliajului electrolitic de Fe-Zn.

30. COLETA DE SABATA, DOROTHEA MIHAILOVICI, RADA BAEA, I.LUMINOSU, Cercetări experimentale asupra influenţei agenţilor exteriori asupra randamentului unui captator cilindric.

31. V.T.DOROBANTU , Funcţia distribuţiei de probabilitate pentru puterea electronică de stopare.

32. D.POPOV , Modelarea potenţialului Morse printr-un potenţial armonic deplasat utilizînd formalismul matricii densităţii.

33. A.ERCUTA , Termometru electronic cu joncţiune p-n.

34. A.ERCUTA , I.MIHALCA , Instalaţie pentru obţinerea curbei de histerezis magnetic.

TRAVAUX DU SEMINAIRE DES MATHEMATIQUE ET DE PHYSIQUE
DE L'INSTITUTE POLYTECHNIQUE "TRAIAN VUIA"DE TIMISOARA
NOVEMBRE 1983

1. DORINA RENDI , SPECIAL TANGENT BUNDLES AND DIFFERENTIABLE MAPS.
2. B.RENDI , ADMISSIBLE SECTIONS IN THE FIBER BUNDLES DEFINITE BY A COMMUTATIVE BANACH ALGEBRA.
3. P.STAVRE, F.C.KLEPP , D-WEYL ISOTHERMIC HYSPERSURFACES AND ASSOCIATED SCALAR INVARIANTS.
4. O.LIPOVAN , σ - ALGEBRAS WITH PROBABILISTIC σ-SUBMEASURES.
5. M.Gh.TUDOR , L'ÉTUDE SUR LA DISTRIBUTION DES ZÉROS DES FONCTIONS ANALYTIQUES (I).
6. N.COFAN, I.STAN, N.ZOPOTA , SUR L'ÉQUIVALANCE DES MÉTHODES REELLES D'INTERPOLATION K ET J PAR LES MÉTHODES DE MOYENNE.
7. IOSEFINA MIHUT , CHERN CLASSES OF FOLIATE MANIFOLDS.
8. D.DAIANU , MORPHISMES SPECIALES EN CATEGORIES FLOUES ET IMMERSIONS (3).
9. N.CRIVAT, T.BANZARU , ON THE QUASI CONTINUITY OF THE LIMITS FOR NETS OF MULTIFUNCTIONS.
10. DOBRINCA MIHAILOV , ON OPERATOR IDEALS ON BANACH COUPLES.
11. ST.MATEI , FUNKTIONEN AUF NUMEROIDALEN STRUKTUREN.
12. GH.BABESCU , REGULARITY AND UNIFORM CONTINUITY PROPERTIES OF COSINE AND SINE CLASS OF OPERATORS.
13. I.GOLET , SUR QUELQUES ÉQUATIONS DANS $\widetilde{\mathcal{C}}_T^S$ - SEMIGROUPS.
14. E.BURLACU, B.CRSTICI, M.NEAGU , ABOUT THE HOMOMORPHISMS OF THE ADDITIVE GROUP $\mathbb{R}$ IN SOME GROUPS OF REALE LINEAR BINARY RELATIONS.
15. P.NASLAU , ON THE REGULATIZATION OF SINGULAR PERTURBATIONS OF SOME LINEAR DIFFERENTIAL EQUATIONS IN CRITICAL CASES.
16. N.NEUHAUS , UBER EIN HYPERGRUPPOID.
17. N.N.NEAMTU , THE TWO-SIDED LAPLACE-DIRICHLET TRANSFORMATION.

18. P.GAVRUTA, T.CEAUSU, ON THE MULTIPLICATION OPERATOR ON FUNCTIONS SPACES.

19. F.RADU, L'ESPACE LINÉAIRE TANGENT GÉNÉRALISÉ D'ORDRE DEUX

20. ELENA TOPUZU , REMARKS ON THE CHARACTERISTIC SPECTRUM OF A DIAGONAL OPERATOR.

21. CARMENCITA UROSU , SUR LES PROPRIÉTÉS LATICEALES DES CONGRUENCES.

22. EMILIA PETRISOR ,ON THE MEASURABILITY OF RANDOM GEODESICS

23. G.HALIC , UTILISATION OF THE NEUTRIE CALCULUS FOR THE INCLUSION OF SOME CLASSES OF FUNCTIONS THAT ARE NOT LOCALLY INTEGRABLE IN THE DISTRIBUTIONS SPACE.

24. COLETA DE SABATA, P.CRACIUN, C.MARCU , ASPECTS CONCERNAND L'EFFET HÉLIO-THERMAL APPLICATION AU STOCAGE DE L'ENERGIE SOLAIRE.

25. COLETA DE SABATA, C.MARCU, I.MIRON, I.LUMINOSU , L'ABSORBTION DE LA RADIATION ÉLECTROMAGNETIQUE PAR UN FERROFLUIDE À BASE DE Fe_3O_4.

26. COLETA DE SABATA, I.MIHALCA, C.MARCU, I.DAMIAN, A.ERCUTA, OPTICAL PROPERTIES OF GLASS CUPOLA USED IN A PYRANOMETER CONSTRUCTION.

27. COLETA DE SABATA, C.MARCU, I.LUMINOSU , ASUPRA DEBITULUI ENERGETIC LA UN HELIOCAPTATOR PLAN FURNIZOR DE AER CALD TEHNOLOGIC.

28. COLETA DE SABATA, C.MARCU, I.DAMIAN, T.PALADE, N.FARBAS , THE DEPENDENCE OF THE STRUCTURE OF Fe-Zn ELECTROLITICAL THIN LAYERS ON THE OBTAINING CONDITIONS.

29. COLETA DE SABATA, C.MARCU, DOROTHEA MIHAILOVICI, I.DAMIAN, MARIA BOLDAN, MARIANA GANGAL, UNTERSUCHUNGEN DER HERSTELLUNGSBEDINGUNGEN UND DER MAGNETISCHEN EIGENSCHAFTEN DER ELEKTROLYTISCH NIEDERGESCHLAGENEN Fe-Zn-LEGIERUNG.

30. COLETA DE SABATA, DOROTHEA MIHAILOVICI, RADA BAEA, I. LUMINOSU, RECHERCHES EXPERIMENTALES SUR L'INFLUENCE DES AGENTS EXTERIEURS AU RENDEMENT D'UN CAPTEURS CYLINDRIQUE.

31. V.T.DOROBANTU , A PROBABILITY DISTRIBUTION FUNCTION FOR THE ELECTRONIC STOPPING POWER.

32. D.POPOV , LE MODELAGE DU POTENTIEL DE MORSE PAR UN POTENTIEL HARMONIQUE DEPLASE PAR L'USAGE DU FORMALISME DE LA MATRICE DE LA DENSITE

33. A.ERCUTA , A p-n JUNCTION ELECTRONIC THERMOMETER.

34. A.ERCUTA , I.MIHALCA, ELECTRONIC HYSTERESIS LOOP TRACER.

Seminarul de Matematică şi Fizică
al
Institutului politehnic "Traian Vuia" Timişoara
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SPECIAL TANGENT BUNDLES AND DIFFERENTIABLE MAPS

Rendi D.

In [5] has studied the special tangent space $T^2_{x_0}(\mathbb{R})$ and differentiable maps, $\varphi : \mathbb{R} \to \mathbb{R}$.

Let M be a C^∞ differentiable manifold of dimension n and $X \in \mathfrak{X}^\infty(M)$ a vector field , $X_p \neq 0 \ \forall p \in M$. We consider the algebra

$$\mathcal{L}_X(p) = \left\{ \hat{f} \mid \hat{f} \in \mathcal{F}_p^\infty,\ f(p) = X_p(\hat{f}) = 0 \right\}$$

and let $L_X^{2n}(p)$ the special tangent space at $p \in M$ [3]. We observe that

$$X_{1,p} = \frac{\partial}{\partial x^2}\Big|_p, \dots, X_{n-1,p} = \frac{\partial}{\partial x^n}\Big|_p, X_{n,p} = \frac{\partial^2}{\partial (x^1)^2}\Big|_p, X_{n+1,p} = \frac{\partial^2}{\partial x^1 \partial x^2}\Big|_p$$

$$\dots, X_{2n-1,p} = \frac{\partial^2}{\partial x^1 \partial x^n}\Big|_p, \ X_{2n,p} = \frac{\partial^3}{\partial (x^1)^3}\Big|_p$$

is a base for $L_X^{2n}(p)$.

Let $\varphi : M \to N$ a C^∞ differentiable map onto N , where M,N are C^∞ differentiable manifolds dim M =n, dim N= k. Let $X \in \mathfrak{X}^\infty(M)$ with $X_p \neq 0 \ \ \forall p \in M$ and $Y \in \mathfrak{Y}^\infty(N)$ with $Y_{\varphi(p)} \neq 0 \ \forall p \in M$. We define the map

$$D_p\varphi : L_X^{2n}(p) \longrightarrow L_Y^{2k}(\varphi(p))$$

by $D_p\varphi(A_p) = B_{\varphi(p)}$, where $B_{\varphi(p)}(\hat{f}) = A_p(\hat{f} \circ \varphi), \forall \hat{f} \in \mathcal{L}_y(\varphi(p))$

Proposition 1. The map $D_p\varphi$ is a linear map (it is called the differential of φ at $p \in M$)

The proof is immediately.

Theorem 1 . The couple $(D\varphi, \varphi)$ where $D\varphi(A_p) = D_p\varphi(A_p)$, $\forall A_p \in L_X^{2n}(p)$ is a homomorphism between the tangent bundles (L_X^{2n}, π, M) and (L_y^{2k}, π', N).

Proof. Let be (U, h_U) a vectorial chart in vector bundles (L_X^{2n}, π, M) which correspond to preferentiale coordinate neighbourhood (U, φ) of M, $(U', h_{U'})$ a vectorial chart in vector bundles (L_Y^{2k}, π', N) which correspond to preferentiale neighbourhood (U', φ') of N. The map $h_{U'}^{-1} \circ D\varphi \circ h_U : U \times \mathbb{R}^{2n} \to U' \times \mathbb{R}^{2k}$ defines the differentiable aplication

$$\Phi_{u,u'}: \varphi^{-1}(u') \cap u \longrightarrow Hom\,(R^{2n}, R^{2k})$$

by

$$(h_{u'}^{-1} \circ D\varphi \circ h_u)(p,z) = (\varphi(p), \Phi_{u',u}(p)\,z)$$

Therefore the theorem is true

Let

$$\mathcal{E}_p^2(M) = \{\hat{f} \mid \hat{f} \in \mathcal{F}_p^\infty(M) \; ; \; f(p) = df_p = 0, \forall f \in \hat{f}\}$$

We can construct the special tangent space $L_2(p)$; [4] $\dim L_2(p) = O_2(n) + O_3(n)$, where $O_i(n)$ is the dimension of linear space of homogeneous polinomials of degree " i ", with n-variables .

The set

$$\left\{ X_{ij,p} = \frac{1}{2!} \frac{\partial^2}{\partial x^i \partial x^j}\Big|_p ; 1 \le i \le j \le n \right\} \cup \left\{ X_{ij,k,p} = \frac{1}{3!} \frac{\partial^3}{\partial x^i \partial x^j \partial x^k}\Big|_p , \right.$$

$\left. 1 \le i \le j \le k \le n \right\}$

form a base for $L_2(p)$ [4]

Let M,N be C^∞ -differentiable manifolds of dimension n, respectivelly k. We consider $\varphi : M \to N$ a C^∞ differentiable map and $L_2^M(p)$, $L_2^N(\varphi(p))$ the corresponding tangent spaces. We can define the differential of φ at $p \in M$, $D_p\varphi : L_2^M(p) \to L_2^N(\varphi(p))$ by $D_p\varphi(X_p)(\hat{f}) = X_p(f \circ \varphi)$, $\forall X_p \in L_2^M(p)$ and $\forall \hat{f} \in \mathcal{E}^2_{\varphi(p)}$ (N)

Proposition 2. The map $D_p\varphi : L_2^M(p) \longrightarrow L_2^N(\varphi(p))$ is a linear map.

The proof is immediately.

To establish the matrix of this map we can consider $D_p\varphi\,(X_{ij,p})$ and $D_p\varphi(X_{ij,k,p})$.

We define the map $D\varphi : L_2^M \to L_2^N$ by $D\varphi(X_p) = D_p\varphi(X_p)$, $\forall X_p \in L_2^M$, where $L_2^M = \bigcup_{p \in M} L_2^M(p)$ and $L_2^N(p) = \bigcup_{q \in N} L_2^N(q)$

Theorem 2. The couple $(D\varphi, \varphi)$ is a homomorfism between the tangent bundles (L_2^M, π, M) and (L_2^N, π', N).

The proof is analogously to theorem 1.

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VECTORI TANGENTI SPECIALI SI APLICATII DIFERENTIALE

Rezumat

In lucrare se studiază comportarea la aplicaţii diferenţiale a spaţiului tangent special $L_X^{2n}(p)$ la o varietate diferenţiabilă cu un cîmp de vectori tangenţi fixat, nenul în orice punct $p \in M$, precum şi a spaţiului tangent special $L_2^M(p)$ la o varietate diferenţiabilă cu singularitate în punct.

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ADMISSIBLE SECTIONS IN THE FIBER BUNDLES DEFINITE BY A COMMUTATIVE BANACH ALGEBRA

Rendi B.

Let A be commutative Banach algebra and M the space of maximal ideals of A. Was construct the fiber bundles $(\overset{\circ}{TM}, \overset{\circ}{\pi}, M)$ and (TM, π, M) with fibres $T_pM = (p/p^2)^*$ and $T_pM = \{D_p \mid D_p : A \to K,$ D_p a derivation on A at point $p \in M\}$

The map $\sigma : M \to TM$ (respectivelly $\sigma : M \to \overset{\circ}{TM}$) is called an admissible section if $\pi \circ \sigma = 1_M$ (respectivelly $\overset{\circ}{\pi} \circ \sigma = 1_M$) and for any $f \in A$ it exists $g \in A$ such that $\sigma(p)(f) = g(p)$ for every $p \in M$ (respectivelly $\sigma(p)([f - f(p)]_{p^2}) = g(p)$ for every $p \in M$).

The isomorphism $\varphi : (TM, \pi, M) \to (\overset{\circ}{TM}, \overset{\circ}{\pi}, M)$ defines a bijection $\Phi : \Gamma(M) \to \overset{\circ}{\Gamma}(M)$ between the spaces of sections

$$\Gamma(M) = \{\sigma \mid \sigma : M \to TM, \pi \circ \sigma = 1_M\}$$

$$\overset{\circ}{\Gamma}(M) = \{\sigma \mid \sigma : M \to \overset{\circ}{TM}; \overset{\circ}{\pi} \circ \sigma = 1_M\}$$

Φ is defines by $\Phi(\sigma) = \varphi \circ \sigma$

Theorem 1. The section $\sigma : M \to TM$ is admissible if and only if $\Phi(\sigma) : M \to \overset{\circ}{TM}$ is an admissible section

Proof. Let $\sigma : M \to TM$ a admissible section.

Is necessary to prove that for every $f \in A$, it exists $g \in A$ such that $(\varphi \circ \sigma)(p)([f - f(p)]_{p^2}) = g(p)$. For $D_p \in T_pM$, $\varphi(D_p) \in (p/p^2)^*$ is defined by $\varphi(D_p)([h]_{p^2}) = D_p(h)$. Is this way $\varphi(\sigma(p))([f - f(p)]_{p^2}) = g(p)$ becomes $\sigma(p)(f - f(p)) = g(p)$.

Since $\sigma(p)$ is a derivation on A at point $p \in M$ we have:

$$\sigma(p)(f - f(p)) = \sigma(p)(f)$$

and σ being admissible section it results that there is $g \in A$ such that $\sigma(p)(f) = g(p)$

Finally $\varphi \circ \sigma$ is an admissible section.

Let $\tau : M \to TM$ an admissible section and let $f \in A$. There exists $g \in A$ such that

$$\tau(p)\left(\left[f-f(p)\right]_{p^2}\right)=g(p)$$

that is

$$(\varphi^{-1}(\tau(p)))(f)=g(p)$$

therefore $\varphi^{-1}\circ\tau$ is an admissible section.

Let $f\in A$ and $p\in M$. The map $df_p: T_pM\to K$ defined by $df_p(D_p)=D_p(f)$ is called the differential of f at $p\in M$. Also we define $d_*f_p : \dot{T}_pM\to K$ by $d_*f_p(T_p)=T_p(\left[f-f(p)\right])$.

We consider the cotangent bundles (TM^*,π^*,M), $(\dot{TM}^*,\dot{\pi}^*,M)$ with fibres $T_pM^*=(T_pM)^*$ and $\dot{T}_pM^*=p/p^2$.

The map $\sigma: M\to TM$ (respectivelly $\sigma: M\to \dot{TM}$) is an admissible section iff $\pi^*\circ\sigma=1_M$ (respectivelly $\dot{\pi}^*\circ\sigma=1_M$) an it exists $f\in A$ such that $\sigma(p)=df_p$ (respectivelly $\sigma(p)=d_*f_p$).

We have an isomorphism

$$\varphi: (TM,\pi,M)\longrightarrow(\dot{TM},\dot{\pi},M)$$

which defines an isomorphism

$$\varphi^*: (\dot{TM}^*,\dot{\pi}^*,M)\longrightarrow(TM^*,\pi^*,M)$$

i.e a bijective map $\varphi^*: \mathring{\Gamma}M^*\longrightarrow\Gamma M^*$ between sets of sections.

$$\mathring{\Gamma}M^*=\{\sigma\mid\sigma: M\to\dot{TM}^*,\ \dot{\pi}^*\circ\sigma=1_M\}$$
$$\Gamma M^*=\{\sigma\mid\sigma: M\to TM^*,\ \pi^*\circ\sigma=1_M\}$$

Theorem 2. The map $\sigma: M\to\dot{TM}^*$ is an admissible section iff $\Phi^*(\sigma)=\varphi^*$ is an admissible section

Proof. Let $\sigma: M\to TM$ an admissible section Hence it exists $f\in A$ such that $\sigma(p)=d_*f_p, \forall p\in M$. Then for $\varphi^*\circ\sigma$ we obtain:

$$(\varphi^*\circ\sigma)(p)(X_p)=(\sigma(p)\circ\varphi)(X_p)=\sigma(p)(\varphi(X_p))=d_*f_p(\varphi(X_p))=$$
$$=\varphi(X_p)(\left[f-f(p)\right])=X_p(f)=df_p(X_p)$$

Therefore $\varphi^*\circ\sigma$ is an admissible section .

Let $\tau: M\to TM^*$ an admissible section and let $\sigma=\Phi^{*-1}(\tau)=(\varphi^{-1})^*\circ\tau$ For $T_p\in T_pM$ we are $((\varphi^{-1})^*\circ\tau)(p)(T_p)=\tau(p)(\varphi^{-1}(T_p))=\tau(p)X_p$, where $X_p=\varphi^{-1}(T_p)$. Since τ is an admissible section it exists $f\in A$ such that $\tau(p)=df_p, \forall p\in M$.

Therefore $((\varphi^{-1})^*\circ\tau)(p)(T_p)=\tau(p)(X_{(p)})=df_p(X_p)=X_p(f)=$
$=\varphi^{-1}(T_p)(f)=T_p(\left[f-f(p)\right])=d_*f_p(T_p)$

Consequently $\Phi^*(\sigma)$ is an admissible section.

Remark. We can consider as cotangent bundle the fiber bundle $((\dot{TM})^*,\dot{\pi}^*,M)$. Since $p/p^2\subseteq(p/p^2)^*$ we can define admissible section for $((\dot{TM})^*,\dot{\pi}^*,M)$ as being an admissible section for

$((\dot{T}M)^*, \dot{\pi}^*, M)$.

Consequently for theory of admissible section we can consider every tangent and cotangent bundles defined in [1], [2], [3].

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SECTIUNI ADMISE IN FIBRATE DEFINITE DE ALGEBRE BANACH COMUTATIVE

Rezumat

In lucrare se studiază legăturile ce există între secţiunile admise în diferite fibrate tangente (respectiv fibrate cotangente) construite în [2], [3].

Se arată că secţiunile admise se corespund.

Seminarul de Matematică şi Fizică
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D-WEYL ISOTHERMIC HYPERSURFACES AND ASSOCIATED SCALAR INVARIANTS

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The study of the unitary fields theory, cosmology, potential theory and many other physical theories, are based on the theory of Riemannian structures $V_n = (M,g,\nabla)$, where $g = (g_{ij})$ is a metric tensor, called the gravitational tensor and ∇ is the corresponding Levi-Civita connection, with the coefficients $\left\{ {i \atop j\ k} \right\}$. In the present paper we consider a structure $L_n = (M,g,\omega,D)$, where M is an n-dimensional differentiable manifold of class C^∞ covered by a system of coordinate neighborhods $\{U,x^k\}$, in which are given a Riemannian metric $g = (g_{ij})$; an 1-form $\omega = (\omega_k)$ called the electromagnetic covector and a connection D, where is semi-symmetrical metrical or is coparallel with ∇. The tensorial and scalar invariants associated to this structure: the concircular invariants [4], or more general the D-concircular invariants [3] and the conharmonic invariants [1] or the D-conharmonic invariants [3], are very important in the above mentioned physical theories. In the following we establish and study two scalar invariants H_1 and H_2 for the $L_n = (M,g,\omega,D)$ structure associated to a family of hypersurfaces Σ.

Definition 1 : A $L_n = (M,g,\omega,D)$ structure, with D a coparallel connection with ∇ relative to the 1-form ω, is called a semi-Weyl structure.

In this case are verified the equations of Weyl typ :

$$g_{jk|r} = 2\omega_r g_{jk} \qquad (1)$$

where $|$ is the covariant derivative relative to D, but these is not symmetric as in the Weyl case, it is semi-symmetric :

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$$T^s_{jk} = \omega_j \delta^s_k - \omega_k \delta^s_j \qquad (2)$$

We consider the conformal transformation of the metric :

$$\overline{g} = e^{2\sigma} g \qquad (3)$$

Noting $p^2 = e^{2\sigma}$ and $p_j = \sigma_{|j}$, we consider the connection D with the components $\overline{\Gamma}$ given by :

$$\overline{\Gamma}^s_{jk} = \Gamma^s_{jk} + p_j \delta^s_k + p_k \delta^s_j - g_{jk} p^s \; ; \quad p^s = g^{si} p_i \qquad (4)$$

It follows easily :

Proposition 1 : The connection $\overline{D}$ defined by (4) is a semi-Weyl connection relative to $\overline{g}$.

We called the transformation $D \to \overline{D}$ or (3) a D-conform transformation. Noting the covariant derivatives relative to $\overline{D}$ with $\|$, we have :

$$\overline{g}_{ij\,\|\,k} = 2\,\overline{\omega}_k\,\overline{g}_{ij} \qquad (\overline{\omega}_k = \omega_k) \; ; \qquad \overline{T}^s_{jk} = T^s_{jk}$$

If we define the tensors :

$$\widetilde{p}_{jk} = p_{j,k} - p_j p_k + \frac{1}{2} g_{jk} p^s p_s \; ; \quad p_{jk} = p_{j|k} - p_j p_k + \frac{1}{2} g_{jk} p^s p_s$$

relative to ∇ and D respectively, then can be declare :

Definition 2 : If $g^{jk}(p_{jk} - p_j \omega_k) = p^s \omega_s$ then the transformation $D \to \overline{D}$ is called an almost conharmonic Weyl transformation.

It follows :

Proposition 2 : The transformation $D \to \overline{D}$ is an almost conharmonic Weyl transformation if and only if : $g^{jk} p_{jk} = p^s \omega_s$.

Remark 1 : In the particular case $\omega = o$ we obtain the conharmonic transformation [3] .

Remark 2 : If p^s and $\omega^s = g^{si}\omega_i$ are orthogonal, then any almost conharmonic Weyl transformation are a conharmonic transformation and reciprocally. In this case the transformation $D \to \overline{D}$ is called a special almost conharmonic Weyl transformation, the electromagnetic vector ω^i being orthogonal to the conform vector p^i .

We consider the hypersurface family : $(\sum) h(x^1, x^2, \ldots, x^n) = c$, where : $h_i = h_{|i}$, $h^k = g^{ks} h_s$; $s^2 = h_i h^i$ relative to g, and the corresponding $\overline{h}^i$, $\overline{s}$ relative to $\overline{g}$. If $\sigma_i = \frac{1}{s} s_i$, then :

$$\underset{1}{H} = \frac{1}{s^2}\left(\nabla^s{}_{|s} + \frac{n-2}{2}\nabla^s\nabla_s\right) \qquad (5)$$

is a scalar invariant of the transformation $D \to \overline{D}$, associated to the family (Σ).

Remark 3 : In the particular case $\omega = o$ the invariant $\underset{1}{H}$ is just the conharmonic invariant $\underset{1}{h}$ [3] .

Remark 4 : If $\omega \neq o$ and $D \to \overline{D}$ is a special almost conharmonical Weyl transformation, then we have simultaneously two scalar invariants $\underset{1}{H}$ and $\underset{1}{h}$ associated to the family (Σ).

We consider the unity vector : $v_i = \frac{1}{s} h_i$, the vector :

$$u_i \overset{def}{=\!=} v_{i|r}v^r - \frac{1}{n-1} v^k{}_{|k} v^i$$

and the corresponding $\overline{v}_i$, $\overline{u}_i$, then we have :

$$g^{ik}v_{i|k} = v^s{}_{|s} + 2\omega_j v^j \; ; \; \overline{g}^{ik}\overline{v}_{i\|k} = \overline{v}^s{}_{\|s} + 2\overline{\omega}_j\overline{v}^j .$$

It follows $\overline{u}_i = u_i - p_i$ analogously with the case of the $\nabla \to \overline{\nabla}$ transformation. We have :

$$u_{i\|j} = u_{i|j} - p_{i|j} - p_i(u_j - p_j) - p_j(u_i - p_i) + g_{ij}p^s(u_s - p_s)$$

$$g^{ij}u_{i|j} = u^k{}_{|k} + 2\omega_s u^s \; ; \; \overline{g}^{ij}\overline{u}_{i\|j} = \overline{u}^k{}_{|k} + \frac{2}{p^2}\omega_s(u^s - p^s)$$

and

$$p^2(\overline{u}^j{}_{\|j} + \frac{n-2}{2}\overline{u}^s\overline{u}_s) = (u^k{}_{|k} + \frac{n-2}{2}u^r u_r) - g^{jk}(p_{jk} - p_j\omega_k) + p^s\omega_s$$

It follows easily :

Theorem 1 : A necessary and sufficient condition that a $D \to \overline{D}$ transformation be an almost conharmonic Weyl transformation is :

$$p^2(\overline{u}^j{}_{\|j} + \frac{n-2}{2}\overline{u}^s\overline{u}_s) = (u^k{}_{|k} + \frac{n-2}{2}u^r u_r) \qquad (6)$$

Since $p^2\overline{s}^2 = s^2$ it follows :

Theorem 2 : An almost conharmonic Weyl transformation $D \to \overline{D}$ has the scalar invariant :

$$\underset{2}{H} = \frac{1}{s^2}\left(u^k{}_{|k} + \frac{n-2}{2}u^s u_s\right) \qquad (7)$$

associated to the family of hipersurfaces (Σ).

Remark 5 : In the particular case $\omega = o$ we obtain the scalar invariant $\underset{2}{h}$ from the conharmonic geometry [3] .

Remark 6 : If $D \to \overline{D}$ is a special almost conharmonical Weyl

transformation, then we have simultaneously two scalar invariants H_2 and h_2 associated to the hypersurfaces family (Σ).

Definition 3 : The hypersurfaces (Σ) are called D-Weyl isothermic, if h is D-Weyl harmonic, t.e. $h^j{}_{|j} = 0$.

In the theory of the constant potential these invariants and theirs properties are important physical interpretations.

The corresponding tensorial invariants being studied by P.Stavre [2].

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HIPERSUPRAFETE D-WEYL IZOTERME SI INVARIANTI SCALARI ASOCIATI

- Rezumat -

In lucrare se consideră o structură $L_n = (M, g, \omega, D)$, care are aplicaţii importante în teoria cîmpurilor fizice şi se stabilesc doi invarianţi scalari asociaţi unei familii de hipersuprafeţe (Σ) în varietatea diferenţiabilă M înzestrată cu această structură, în cazul unei transformări conforme: $\bar{g} = e^{2\sigma} g$. Se constată că în cazul particular $\omega = 0$, cei doi invarianţi stabiliţi H_1 şi H_2 se reduc la invarianţii coarmonici h_1 şi h_2 stabiliţi de P.Stavre. Se defineşte şi noţiunea de hipersuprafaţă D-Weyl izotermă, care are aplicaţii în teoria cîmpurilor fizice.

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σ-ALGEBRAS WITH PROBABILISTIC σ-SUBMEASURES

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The notion of $\mathcal{B}$-algebra with probabilistic (Menger) submeasure was defined in [1]. In whath follows this notion is extended for the case of σ-algebras with probabilistic σ-subadditive submeasures, presenting in a topological manner the classical procedure of completing a σ-algebra by the negligible sets adjunction. Also, it is shown that a probabilistic σ-submeasure defined on a σ-algebra, induces on $\mathcal{B}$ a complete Menger type probabilistic semimetric space structure.

Let T be a t-norm i.e. an operation on $[0,1]$ such that $([0,1],T)$ is an order semigroup with unit 1. We suppose that $T(a,b) \geqslant \mathrm{Max}(a+b-1, 0)$. In the probabilistic analysis, the t-norm T is used instead of the sum from the theory of classical structures (for exemple in the case of metric probabilistic spaces) and so it is natural to define operators which replace the finite sums of n elements and the infinite sums.

Let $T^n(a_1,a_2,\ldots,a_n) = T(a_1,T(a_2,\ldots,T(a_{n-1},a_n)\ldots))$ and $T(a_1,a_2,\ldots) = \lim\left[T^n(a_1,a_2,\ldots)\right]$. We denote also by $\overset{n}{\underset{k=1}{T}} a_k$ and respectively $\overset{\infty}{\underset{k=1}{T}} a_k$ these operators.

Let X be a non-empty set, $\mathcal{B} \subset \mathcal{P}(X)$ an σ-algebra. In [1] and [6] the following concept has been defined:

Definition 1. We state that the map $\gamma : \mathcal{B} \longrightarrow \Delta$ is a probabilistic submeasure on $\mathcal{B}$ with respect to the t-norm T if :

(M.P.I) $\gamma_\phi = H$

(M.P.II) $A \subset B$ imply $\gamma_A(x) \geqslant \gamma_B(x)$ for every $x > 0$, and $A, B \in \mathcal{B}$

(M.P.III) for each A, B in $\mathcal{B}$:

$$\gamma_{A\cup B}(x+y) \geqslant T(\gamma_A(x), \gamma_B(y)) \quad , x > 0 \ , \ y > 0.$$

In the sequel we shall call probabilistic σ-submeasure on $\mathcal{B}$

BUDAPESTI MUZSAKI

with respect to the t-norm T, a probabilistic submeasure which verifies the condition

(M.P.III') For every $\{x_n\}_{n\in N}$, $x_n > 0$, $\sum_{n=1}^{\infty} x_n < \infty$, and every $\{A_n\}_{n\in N} \subset \mathcal{B}$ let's have:

$$\gamma_{\bigcup_{n=1}^{\infty} A_n}\left(\sum_{n=1}^{\infty} x_n\right) \geq \prod_{n=1}^{\infty} \gamma_{A_n}(x_n)$$

where: $\prod_{n=1}^{\infty} \gamma_{A_n}(x_n) = \lim_{n\to\infty} T(\gamma_{A_1}(x_1), T(\gamma_{A_2}(x_2), \ldots T(\gamma_{A_{n-1}}(x_{n-1}), \gamma_{A_n}(x_n))\ldots)$.

Particularly : (M.P.III') ==> (M.P.III)

The triplet $(\mathcal{B}, \gamma, T)$ is called the σ-algebra with probabilistic (or Menger) σ-submeasure. In this paper we will consider only the finite probabilistic submeasure, i.e. submeasure such that : $\sup_{x>0} \gamma_A(x) = 1$ for every $A \in \mathcal{B}$.

If $\gamma : \mathcal{B} \longmapsto \Delta$, is a finite probabilistic σ-submeasure with respect to continuous t-norm T, then the outer σ-submeasure induced by γ, denoted γ^* is defined by: $\gamma^* : \mathcal{P}(X) \longrightarrow \Delta$, where:

$\gamma^*_A(x) = \sup \{ \gamma_E(x) ; A \subseteq E \in \mathcal{B} \}$, $A \subset X$.

It is known that if the t-norm T is continuous, the family $\{ U(\varepsilon,\lambda) : \varepsilon > 0, \lambda > 0 \}$ where $U(\varepsilon,\lambda) = \{ A \subset X ; \gamma^*_A(\varepsilon) > 1-\lambda \}$ is a normal base of neighbourhoods of ϕ, for a Fréchet-Nikodym topology τ_γ and $(\mathcal{P}(X), \tau_\gamma) = \gamma[\mathcal{P}(X)]$ is a topological algebra [1].

We denote by $\gamma(\mathcal{B})$ the subalgebra $\mathcal{B}$ of the topologicaly ring $\gamma(\mathcal{P}(X))$.

Definition 2. A set $N \subseteq X$ will be called γ-null if $\gamma^*_N(x) = H(x)$, $x > 0$. We denote $\mathcal{N}_\gamma = \{N ; N \subseteq X, N \text{ is } \gamma\text{-null}\}$

Theorem 1. $\mathcal{N}_\gamma = \overline{\{\phi\}}$, where $\overline{\{\phi\}}$ is the adherence of $\{\phi\}$ in $\gamma(\mathcal{P}(X))$. The proof follows directly from definitions.

The following theorem describes in a topological manner the classical procedure of completing a σ-algebra $\mathcal{B}$ by the γ-null sets adjunction.

Theorem 2. Let $(\gamma, \mathcal{B}, T)$ the σ-algebra with T-probabilistic σ-submeasure. Then :

$$\mathcal{B} \triangle \mathcal{N}_\gamma = \overline{\mathcal{B}}$$

where $\overline{\mathcal{B}}$ is the adherence of $\mathcal{B}$ in $\gamma(\mathcal{P}(X))$.

Proof.

For $\varepsilon > 0$, $\lambda > 0$, we denote $U_{(\varepsilon,\lambda)} = \{ E \subset X ; \gamma^*_E(\varepsilon) > 1-\lambda \}$

If $E\in\mathcal{B}$ and $N\in\mathcal{N}_\gamma$, the sets $(E\triangle N)\mathring{\triangle}\bigcup(\varepsilon,\lambda)$ forms a fundamental system of neighbourhoods of $E\triangle N$ in $\gamma(\mathcal{P}(X))$. But, for every $\varepsilon>0$, $\lambda>0$, $N\in\bigcup(\varepsilon,\lambda)$ and $E\in(E\triangle N)\mathring{\triangle}\bigcup(\varepsilon,\lambda)$ which shows that $\mathcal{B}\mathring{\triangle}\mathcal{N}_\gamma\subset\overline{\mathcal{B}}$. Reciprocally, let $A\in\overline{\mathcal{B}}$, $\varepsilon>0$, and $x>0$. Because T is continuous in (1,1), and $\lim_{a,b\to 1}T(a,b)=1$, for every $\varepsilon>0$ there exists ε_1 and λ_1 such that $T(1-\varepsilon_1,1-\lambda_1)>1-\varepsilon$. For $1-\lambda_1$ there exists ε_2 and λ_2 such that $T(1-\varepsilon_2,1-\lambda_2)>1-\lambda_1$. In this manner, a sequence $\{\varepsilon_n\}_{n\in N}$ is determined, such that $\prod_{n=1}^{\infty}(1-\varepsilon_n)>1-\varepsilon$. For $n=1.2,\dots$, $[A\mathring{\triangle}\bigcup(x,\varepsilon_n)]\cap\mathcal{B}\neq\phi$, there fore $U_n\in\bigcup(\frac{x}{2^n},\varepsilon_n)$ and $E_n\in\mathcal{B}$ such that $A\triangle U_n=E_n$. From the inclusions $A-U_n\subseteq A$, $U_n-A\subseteq X-A$, we have

$$\lim_{n\to\infty}\sup E_n=\bigcap_{n=1}^{\infty}\bigcup_{k\geq n}[(A-U_k)\cup(U_k-A)]=\bigcap_{n=1}^{\infty}\bigcup_{k\geq n}(A-U_k)\cup\bigcap_{n=1}^{\infty}\bigcup_{k\geq n}(U_k-A)=$$

$$=[A\cap\bigcap_{n=1}^{\infty}\bigcup_{k\geq n}(X-U_k)]\cup[(\bigcap_{n=1}^{\infty}\bigcup_{k\geq n}U_k)-A]=[A-(\lim_{n\to\infty}\inf U_n)]\triangle[(\lim_{n\to\infty}\sup U_n)-A],$$

and since:

$$A=[A-(\lim_{n\to\infty}\inf U_n)]\triangle[A\cap\lim_{n\to\infty}\inf U_n)],\text{ it rezults}$$

$$A=\lim_{n\to\infty}\sup E_n\triangle[(\lim_{n\to\infty}\sup U_n)-A]\triangle[A\cap(\lim_{n\to\infty}\inf U_n)]$$

But:

$$\gamma^*\Big(\bigcup_{k\geq n}U_k\Big)\Big(\sum_{k=n}^{\infty}\frac{x}{2^{k-n+1}}\Big)\geq\prod_{i=n}^{\infty}\Big(\gamma_{U_i}^*\Big(\frac{x}{2^{i-n+1}}\Big)\Big)\geq\prod_{i=1}^{\infty}(1-\varepsilon_i)>1-\varepsilon,\ \forall\varepsilon>0$$

Then:

$$\gamma^*_{[(\limsup U_n)-A]}(x)=H(x)=\gamma^*_{[A\cap\liminf U_n]}(x),\quad x>0$$

There fore $A\in\mathcal{B}\mathring{\triangle}\mathcal{N}_\gamma$ since $\mathcal{B}$ is a σ-algebra.

Theorem 3. If $(\mathcal{B},\gamma,T)$ is a σ-algebra with probabilistic σ-submeasure, then $\gamma(\mathcal{B})$ is complete semi-Menger space, with respect to the probabilistic semimetric $\mathcal{F}:\mathcal{B}\times\mathcal{B}\longrightarrow\Delta$, $\mathcal{F}(A,B)=F_{AB}=\gamma_{A\triangle B}$.

Proof. The (ε,λ) uniformity on $\mathcal{B}$ induced by probabilistic semimetric $\mathcal{F}$ and the uniformity on $\mathcal{B}$ induced by the topology $\gamma(\mathcal{B})$ are identical.

To prove the completeness of $\gamma(\mathcal{B})$, let $\{E_n\}_{n=1}^{\infty}$ there be a Cauchy sequence in $\gamma(\mathcal{B})$. Then there exists a subsequence $\{E_{k_n}\}_{n=1}^{\infty}$, so that for every $\varepsilon>0$, $x>0$, there exists a $n_0\in N$, so that for $n\geq n_0$, $\gamma_{E_{k_n}\triangle E_{k_{n+1}}}(\frac{x}{2^n})>1-\varepsilon_n$.

The sequence $\{\varepsilon_n\}_{n\in N}$ is chosen such that $\prod_{n=1}^{\infty}(1-\varepsilon_n)>1-\varepsilon$.

Let $B=\lim_{n\to\infty}\sup E_{k_n}\in\mathcal{B}$. We will show that :

$$\mathcal{F}(E_{k_n},B)(x) = \gamma_{E_{k_n}\Delta B}(x) \longmapsto H(x)\ ,\quad x>0\ ,$$

from where it results that $\gamma(\mathcal{B})$ is complet .

We note $A_n = E_{k_n} \Delta E_{k_{n+1}}$ .From the relation $E_{k_n} \Delta B \subset \bigcup_{k\geq n} A_k$, it rezults :

$$\gamma_{E_{k_n}\Delta B}(x) \geq \gamma_{\bigcup_{k\geq n} A_k}\left(\sum_{k\geq n} \frac{x}{2^{k-n+1}}\right) \geq \prod_{i=n}^{\infty}(1-\varepsilon_i) > \prod_{i=1}^{\infty}(1-\varepsilon_i) > 1-\varepsilon\ , \text{ for } n \geq n_0,$$

which proves the assertion.

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σ-ALGEBRE CU σ-SUBMĂSURI PROBABILISTE

Rezumat

In lucrare se defineşte noţiunea de σ-algebră cu submăsură probabilistă σ-subaditivă,prezentînd într-o manieră topologică procedeul clasic de completare a unei σ-algebre prin adjuncţia mulţimilor neglijabile.

Se arată de asemenea că o σ-submăsură probabilistă definită pe o σ-algebră $\mathcal{B}$,determină pe $\mathcal{B}$ o structură de spaţiu semimetric de tip Menger complet.

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L'ÉTUDE SUR LA DISTRIBUTION DES ZÉROS DES FONCTIONS ANALYTIQUES (I)

Tudor M.Gh.

Soit $\mathcal{H}^+$ l'ensemble des fonctions de la forme

(1) $f(z)=\sum_0^\infty a_n z^n$, $a_n \in R_+$, $|z|<R_f$ -(R_f-le rayon de convergence) et des polynômes ayant comme coefficients, les nombres positifs.

Dans ce travail, nous nous proposons de determiner une limite inférieure pour les modules des zéros des quelques fonctions de la classe $\mathcal{H}^+$ et dans ce but nous allons démonstrer un théorème générale. Pour cela, nous introduisons les fonctionnelles suivantes: $\omega_s : \mathcal{H}^+ \longrightarrow R_+^\circ$, $s=0,1,2$, ainsi:

(2) $\omega_0[f]=\inf_{n\geq 1}\frac{na_n}{(n+1)a_{n+1}}$, $\omega_1[f]=\min\left(\frac{a_0}{a_1},\omega_0[f]\right)$, $\omega_2[f]=\min\left(\frac{a_0}{2a_1},\omega_0[f]\right)$

qui satisfont les inégalités: $\omega_0[f]\geq\omega_1[f]\geq\omega_2[f]\geq 0$

Lemme 1. Si $f_1,f_2,\dots,f_p\in\mathcal{H}^+$ et $\alpha_1,\alpha_2,\dots,\alpha_p$ sont réels positifs, alors on a lieu l'inégalité suivante:

(3) $\omega_s\left[\sum_1^p \alpha_i f_i\right]\geq \min_{i=\overline{1,p}}(\omega_s[f_i])$, $s=0,1,2$

Démonstration. Il suffit d'utiliser l'inégalité connue:

(4) $\min_{i=\overline{1,p}}(a_i/b_i)\leq\left(\sum_1^p\lambda_i a_i\right)/\left(\sum_1^p\lambda_i b_i\right)\leq\max_{i=\overline{1,p}}(a_i/b_i)$

qui est vraie pour tous systèmes de nombres positifs:

$(a_1,a_2,\dots,a_p)$, $(b_1,b_2,\dots,b_p)$, $(\lambda_1,\lambda_2,\dots,\lambda_p)$

Lemme 2. Si $f_1, f_2,\dots, f_p\in\mathcal{H}^+$, alors il existe l'inégalité

(5) $\frac{1}{\omega_s[f_1f_2\dots f_p]}\leq\min\left(\frac{1}{\omega_s[f_{i_1}]}+\frac{1}{\omega_2[f_{i_2}]}+\dots+\frac{1}{\omega_2[f_{i_p}]}\right)$

ou min est consideré par rapport à tous permutations $(i_1,i_2,\dots,i_p)$ des nombres $1,2,\dots,p$.

Démonstration. Soit d'abord le cas de deux fonctions:

$f(z)=\sum_0^\infty a_n z^n$, $g(z)=\sum_0^\infty b_n z^n$, $h(z)=f(z)g(z)=\sum_0^\infty c_n z^n$

En utilisant l'inégalité (4) nous avons succesivement:

$$\frac{c_0}{C} = \frac{t a_0 b_0 + (1-t) a_0 b_0}{a_0 b_1 + a_1 b_0} \geq \min\left(t\frac{a_0}{a_1}, (1-t)\frac{b_0}{b_1}\right) \geq \min\left(t\omega_1[f], (1-t)\omega_1[g]\right)$$

$$\frac{n}{n+1}\frac{c_n}{c_{n+1}} \geq \min\left(t\frac{n}{n+1}\frac{a_n}{a_{n+1}}, (1-t)\frac{n}{n+1}\frac{b_0}{b_1}, \frac{n}{n+1}\frac{b_1}{b_2}, \dots \frac{n}{n+1}\frac{b_n}{b_{n+1}}\right)$$

$$\geq \min\left(t\omega_0[f], (1-t)\omega_2[g]\right), \quad n=1,2,\dots, \quad (\forall)\ t \in [0,1]$$

Ainsi que, nous déduison les inégalités suivantes:

(6) $\omega_s[fg] \geq \min\left(t\omega_s[f], (1-t)\omega_2[g]\right)$, $s=0,1; t\in[0,1]$

Pour l'optimisation de l'inégalité (6) quand $t\in[0,1]$ il est nécessaire de determiner un point $t_0 \in [0,1]$ pour quel on a $\sup\min_{t\in[0,1]}\left(t\,\omega_s[f], (1-t)\,\omega_2[g]\right)$. On obtient ,

(7) $$\frac{1}{\omega_s[fg]} \leq \frac{1}{\omega_s[f]} + \frac{1}{\omega_2[g]}, \quad s=0,1$$

Par permutation $f \rightleftarrows g$ on deduit l'inégalité

(8) $$\frac{1}{\omega_s[fg]} \leq \min\left(\frac{1}{\omega_s[f]} + \frac{1}{\omega_2[g]}, \frac{1}{\omega_s[g]} + \frac{1}{\omega_2[f]}\right)$$

De l'inégalité (8) on déduit l'inégalité (5), par l'association des facteurs du produit $f_1, f_2, \dots, f_p$

Lemme 3. Soient les systèmes de fonctions de $\mathcal{H}^+$, $(f_1^i, f_2^i, \dots, f_{p_i}^i)$, $i=1,2,\dots,m$.

On a lieu l'inégalité:

(9) $$1\Big/\left(\sum_{i=1}^{m}\prod_{k=1}^{p_i} f_k^i\right) \leq \max_{i=\overline{1,m}}\left\{\min\left(\frac{1}{\omega_s[f_{k_1}^i]} + \frac{1}{\omega_2[f_{k_2}^i]} + \dots \frac{1}{\omega_2[f_{k_{p_i}}^i]}\right)\right.$$

dans lequelle min est consideré par rapport à tous permutations $(k_1, \dots, k_{p_i})$ des mombres $(1,2,\dots,p_i)$, indépendantes de sommes du membre droit.

Pour la démonstration, s'appliquent les lemmes 1 et 2

Lemme 4. Si $f \in \mathcal{H}^+$, alors f ne s'annule pas, dans le domain $|z| < \omega_1[f]$ et f n'admet pas des zéros complexes dans le domaine $|z| < \omega_0[f]$.

Démonstration. En notant $z = re^{i\varphi}$, nous avons :

(10) $\mathrm{Ref}(z) = \lim_{n\to\infty} K_n(r, \varphi, \bar{a})$, $\mathrm{Im} f(z) = \lim_{n\to\infty} I_n(r, \varphi, \bar{a})$

où ont été utilisées les notations:

$$K_n(r,\varphi,\bar{a}) = \delta_0(r) + \sum_{\nu=1}^{n-1} \delta_\nu(r)\sigma_\nu^*(\varphi)r^\nu + n a_n r^n \sigma_n^*(\varphi)$$

$$I_n(r,\varphi,\bar{a}) = \sum_{\nu=1}^{n-1} \delta_\nu(r)\sigma_\nu(\varphi)r^\nu + n a_n r^n \sigma_n(\varphi)$$

$$\delta_0(r) = a_0 - a_1 r,\ \delta_\nu(r) = \nu a_\nu - (\nu+1)a_{\nu+1} r,\ \sigma_n(\varphi) = \sum_1^n \frac{\sin\nu\varphi}{\nu},\ \sigma_n^*(\varphi) = 1 + \sum_1^n \frac{\cos\nu\varphi}{\nu}$$

En tenant compte de les inégalités connues [1]

$\sigma_n(\varphi) > 0$, $\sigma_n^*(\varphi) > 0$, $\varphi \in]0,\pi[$, $n = 1,2,\dots$

et en posant les conditions $\delta_\nu(r) > 0$, $\nu = 1,2,\dots$ on peut observer que, dans ce cas $\mathrm{Im} f(z) > 0$ et donc f n'admet pas des zéros complexes dans le domaine $|z| < \inf_{n\geq 1}(n a_n / (n+1)a_{n+1}) =$

$= \omega_0[f]$.

En utilisant l'expression du $K_n(r,\varphi,\bar{a})$, on montre analoguement que f ne s'annule pas dans le domaine $|z| < \omega_1[f]$.

Théorème. Soient les systemes de fonctions de $\mathcal{H}^+$, notées $(f_1^i, f_2^i, \dots, f_{n_i}^i)$, $i = \overline{1,m}$, et soit la fonction

11) $$F(z) = \sum_{i=1}^{m} \prod_{k=1}^{p_i} f_k^i(z),\quad 0 \leq |z| < R_F \leq +\infty$$

La fonction F(z) ne s'annule pas dans le domaine $|z| < R_1$ et n'admet pas des zeros complexes dans le domaine $|z| < R_0$ où

12) $$R_s = 1\Big/\left(\max_{i=\overline{1,m}}\left\{\min\left(\frac{1}{\omega_s[f_{k_1}^i]} + \frac{1}{\omega_2[f_{k_2}^i]} + \dots + \frac{1}{\omega_2[f_{k_{p_i}}^i]}\right)\right\}\right),\ s = 0,1$$

Demonstration.

En appliquant la lemme 4, il en résulte que F(z) ne s'annule pas pour $|z| < \omega_1[F]$ et n'admet pas des zeros complexes dans le domaine $|z| < \omega_0[F]$.

En appliquant la lemme 3, on déduisent les inégalités

$$\omega_1[F] \geq R_1,\quad \omega_0[F] \geq R_0$$

Donc, F ne s'annnule pas pour $|z|<R_1$ et n'admet pas des zéros complexes pour $|z|<R_0$

Remarque.

Pour un polynome $P_n(z) \in \mathcal{H}^+$ on déduit:

$$\omega_1[P_n(z)] \leq |z_0| \leq \frac{1}{\omega_1[z^n P_n(\frac{1}{z})]} \tag{13}$$

z_0-zéro quelconque

$$\omega_0[P_n(z)] \leq |z_0'| \leq \frac{1}{\omega_0[z^n P_n(\frac{1}{z})]} \tag{14}$$

z'_0- zéro complex

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STUDIU ASUPRA DISTRIBUTIEI ZEROURILOR FUNCTIILOR ANALITICE

Rezumat

In lucrare se determină limite inferioare pentru modulele zerourilor funcţiilor de forma $f(z) = \sum_0^\infty a_n z^n$ cu $a_n > 0$, $n = 0,1,2,\ldots$ (sau polinoame) şi pentru anumite combinaţii de astfel de funcţii.

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SUR L'ÉQUIVALENCE DES METHODES REÉLLES D'INTERPOLATION K ET J PAR LES METHODES DE MOYENNE.

Cofan N., Stan I., Zopota N.

Dans cet article on etabli les conditions dans les quelle les espaces d'interpolation obtenus par les méthodes K et J coincident (avec des normes équivalentes) avec les éspaces obtenus avec les méthodes de moyenne. Pour des détails on peut voir [1] .

Soit A_0, A_1 un comple Banach, ρ_0 et ρ_1 deux normes invariantes par rearengement , et φ_{ϑ_0} , φ_{ϑ_1} deux nombres reéls tels que $\varphi_{\vartheta_0} \cdot \varphi_{\vartheta_1} < 0$.

Notous par

$$G_0 = \{ x \in R : (\varphi_{\vartheta_1} - \varphi_{\vartheta_0})x \geqslant 0 \}$$

$$G_1 = \{ x \in R : (\varphi_{\vartheta_0} - \varphi_{\vartheta_1})x \geqslant 0 \}$$

G_0 et G_1 donnent une division de l'espace R.

Definition 1. Soit A un espace Banach, $\varphi_\vartheta \in R$ et ρ une norme invariante par rearangement On définit l'ensemble $L^{\rho}_{\varphi_\vartheta}(A)$ par:

$$L^{\rho}_{\varphi_\vartheta}(A) = \{ u : R \to A \text{ fortement mésurables}, \rho(e^{\varphi_\vartheta x} \| u(x) \|_A) < \infty \}$$

Proposition 1. L'application

$$L^{\rho_0}_{\varphi_{\vartheta_0}}(A_0) \cap L^{\rho_1}_{\varphi_{\vartheta_1}}(A_1) \ni U(x) \longrightarrow \int_R u(x)\, dx \in A_0 + A_1$$

est linéaire et cántinue.

Démonstration. La propriéte suivante a lieu:

$$\int_R \| u(x) \|_{A_0+A_1} dx = \int_{G_0} \| u(x) \|_{A_0+A_1} dx + \int_{G_1} \| u(x) \|_{A_0+A_1} dx$$

$$\leqslant \text{const} \left(\int_{G_0} \| u(x) \|_{A_0} dx + \int_{G_1} \| u(x) \|_{A_1} dx \right) .$$ On utilisant l'inégalité de Hölder et sachant que $\int_{G_i} e^{-\varphi_i x} dx < \infty$ ([2]) on obtient que:

$$\int_{G_i} \| u(x) \|_{A_j} dx = \int_{G_i} e^{-\varphi_i x} e^{\varphi_i x} \| u(x) \|_{A_j} dx \leq \rho_j'\left(e^{\varphi_i x} \chi_{G_j} \right) \rho_j\left(e^{\varphi_i x} \| u(x) \|_{A_j} \right) < \infty$$

où ρ_j' est la norme associée à ρ_j.

Definition 2. ([3]) On note par $S(\rho_0, \varphi_{\beta_0}, A_0 ; \rho_1, \varphi_{\beta_1}, A_1)$ l'espace.

$$S(\rho_0,\varphi_{\beta_0},A_0;\rho_1,\varphi_{\beta_1},A_1)=\Big\{a\in A_0+A_1 : \exists u\in\bigcap_{i=0}^{1} L^{\rho_i}_{\varphi_{\beta_i}}(A_i),\ a=\int_R u(x)\,dx\Big\}$$

C'est facile de verifier que $S = S(\rho_0, \varphi_{\beta_0}, A_0 ; \rho_1, \varphi_{\beta_1}, A_1)$ avec la norme

$$\|a\|_S = \inf\Big\{\max\Big[\rho_0\big(e^{\varphi_{\beta_0}t}\|u(t)\|_{A_0}\big), \rho_1\big(e^{\varphi_{\beta_1}t}\|u(t)\|_{A_1}\big)\Big],\ a=\int_R u(x)\,dx\Big\}$$

forme un éspace Banach. S est espace d'interpolation de type $\theta = \varphi_{\beta_0}/(\varphi_{\beta_0}-\varphi_{\beta_1})$ ([3])

Proposition 2 ([3]) $S(\rho_0, \varphi_{\beta_0}, A_0 ; \rho_1, \varphi_{\beta_1}, A_1) = S(\rho_0, \theta, A_0 ; \rho_1, 1-\theta, A_1)$

où $\theta = \varphi_{\beta_0}/(\varphi_{\beta_0}-\varphi_{\beta_1})$.

On considère l'espace

$$\underline{S}(\rho_0,\varphi_{\beta_0},A_0;\rho_1,\varphi_{\beta_1},A_1)=\Big\{a\in A_0+A_1;\ \exists v_i\in L^{\rho_i}_{\varphi_{\beta_i}}(A_i),\ a=v_0(t)+v_1(t)\ p.p.\ \mu\Big\}$$

Cet espace, par rapport aux normes equivalentes

$$\|a\|_{\underline{S}} = \inf\Big\{\max\Big[\rho_0\big(e^{\varphi_{\beta_0}t}\|v_0(t)\|_{A_0}\big), \rho_1\big(e^{\varphi_{\beta_1}t}\|v_1(t)\|_{A_1}\big)\Big],\ a=v_0(t)+v_1(t)\Big\}$$

et

$$\|a\|_{\underline{S}} = \inf\Big\{\rho_0\big(e^{\varphi_{\beta_0}t}\|v_0(t)\|_{A_0}\big)+\rho_1\big(e^{\varphi_{\beta_1}t}\|v_1(t)\|_{A_1}\big);\ a=v_0(t)+v_1(t)\Big\}$$

est un espace Banach.

Dans [2] on étudie les méthodes reelles K et J d'interpolation. Si on considère $\rho = (\rho_0, \rho_1)$ et on définit les espaces $A_{\theta,\rho,J}$, $A_{\theta,\rho,K}$ de la maniere suivante:

$$a\in A_{\theta,\rho,J} \Leftrightarrow \|a\|_{\theta,\rho,J} = \inf\Big[\max_{k=0,1}\big(\rho_k\big(t^{k-\theta}\|u(t)\|_{A_k}\big)\big)\Big]$$

et

$$a\in A_{\theta,\rho,J} \Leftrightarrow \|a\|_{\theta,\rho,K} = \inf\Big\{\rho_0\big(t^{-\theta}\|a_0(t)\|_{A_0}\big)+\rho_1\big(t^{1-\theta}\|a_1(t)\|_{A_1}\big),\ a=a_0(t)+a_1(t)\Big\}$$

alors le théorème suivant a lieu

Théorème 1. Les proposition suivantes sont equivalentes:

1) $a \in A_{\theta,\rho,J}$

2) $a\in S(\rho_0,-\theta,A_0;\rho_1,1-\theta,A_1)$

3) $a\in \underline{S}(\rho_0,-\theta,A_0;\rho_1,1-\theta,A_1)$

4) $a\in A_{\theta,\rho,K}$

Démonstration. Soit $a \in A_{\theta,\rho,J}$ alors il existe $u:(0,\infty)\to A_0\cap A_1$ fortement mésurable telle que $\rho_i\big(t^{-\theta}\|u(t)\|_{A_i}\big)<\infty$ $(i=0,1)$

et $a = \int_0^\infty u(t)\,dt/t$. On utilisant le changement de variable $t=$ on obtient $a = \int_R v(s)ds$ et $\rho_i\left(e^{(i-\theta)s}\|v(s)\|_{A_i}\right) = \rho_i\left(t^{i-\theta}\|u(t)\|_{A_i}\right)$ $(i=$

donc $a \in S(\rho_0, \xi_0, A_0 ; \rho_1, \xi_1, A_1)$

Si $a \in S$, alors il existe $u \in \bigcap_{i=0}^{1} L^{\rho_i}_{\xi_i}(A_i)$ tel que $a = \int_{\mathbb{R}} u(x)\,dx$. On posant $v_i(x) = \int_{-G_i} u(x-y)dy$ alors on obtient $a = v_0(x) + v_1(x) = \int_R u(x-y)dy$. Parce que $e^{\xi_i x} v_i(x) = \int_{-G_i} e^{\xi_i y} e^{\xi_i(x-y)} u(x-y)dy$ on obtient que

$$e^{\xi_i x}\|v_i(x)\|_{A_i} \leq \int_{G_i} e^{\xi_i y} e^{\xi_i(x-y)}\|u(x-y)\|_{A_i}dy = \int_{\mathbb{R}} f_i(y)\,g_i(x-y)\,dy$$

où $f_i(y) = \begin{cases} e^{\xi_i y}, & y \in -G_i \\ 0, & \text{en reste} \end{cases}$, $g_i(y) = e^{\xi_i y}\|u(y)\|_{A_i}$

d'où il résulte que $v_i \in L^{\rho_i}_{\xi_i}(A_i)$

On sachant que $f_i \in L^1$ et $g_i \in L^{\rho_i}$ et on appliquant l'inégalité de Young([1]) on obtient:

$$\rho_i\left(e^{\xi_i x}\|v_i(x)\|_{A_i}\right) \leq \|f_i\|_{L_1}\rho_i\left(e^{\xi_i y}\|u(y)\|_{A_i}\right) < \infty \quad (i=0,1)$$

donc $a \in \underline{S}$.

Soit $a \in \underline{S}$. Donc il existe $v_i \in L^{\rho_i}_{\xi_i}(A_i)$ tel que $a = v_0(s) + v_1(s)$ p.p.μ et $\rho_k\left(e^{(k-\theta)s}\|v_k(s)\|_{A_k}\right) < \infty$ $(k=0,1)$

On utilisant le changement de variable $s = \ln t$ on obtient $a \in A_{\theta,\rho,K}$.

Soit $a \in A_{\theta,\rho,K}$. D'aprés le lemme fundamentale de l'interpolation, il existe $u:(0,\infty) \longrightarrow A_0 \cap A_1$ fortement mesurable telle que $a = \int_0^\infty u(t)dt/t$ et $J(t,u(t)) \leq 4eK(t,a) \Longleftrightarrow$

$$\max(\|u(t)\|_{A_0}\ t(\|u(t)\|_{A_1}) \leq 4e \inf\left\{\|a_0(t)\|_{A_0} + t\|a_1(t)\|_{A_1}\right\}$$

d'où

$$t^{-\theta}\|u(t)\|_{A_0} \leq 4e\,t^{-\theta}\|a_0(t)\|_{A_0},\quad t^{1-\theta}\|u(t)\|_{A_1} \leq 4e\,t^{1-\theta}\|a_1(t)\|_{A_1}$$

et en utilisant la monotonie de la normes ρ_k $(k=0,1)$

$$\rho_0\left(t^{-\theta}\|u(t)\|_{A_0}\right) \leq 4e\,\rho_0\left(t^{-\theta}\|a_0(t)\|_{A_0}\right)$$
$$\rho_1\left(t^{1-\theta}\|u(t)\|_{A_1}\right) \leq 4e\,\rho_1\left(t^{1-\theta}\|a_1(t)\|_{A_1}\right)$$

donc $a \in A_{\theta,\rho,J}$.

L'equivalence des normes rezultes immediatement dé la démonstration du théorème 1.

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ASUPRA ECHIVALENTEI METODELOR REALE DE INTERPOLARE K şi J CU METODELE DE MEDIE

Rezumat

In această lucrare se stabilesc condiţiile în care spaţiile de interpolare , obţinute cu metodele K şi J , coincid cu cele obţinute cu metodele de medie.

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CHERN CLASSES OF FOLIATE MANIFOLDS

Mihuţ I.

Definition 1. A complex foliated structure $\mathcal{F}$ analytical along the leaves , of (complex) codimension n of the n+m-dimensional paracompact complex variety V is given by an atlas $\{U_\alpha , z^a_\alpha , z^b_\alpha\}$, a,b= 1,...,n, u,v= n+1,...,n+m such that on $U_\alpha \cap U_\beta \neq \emptyset$

$$\frac{\partial z^a_\beta}{\partial z^u_\alpha} = 0 \quad \text{and} \quad \frac{\partial z^u_\alpha}{\partial \bar{z}^a_\alpha} = 0$$

A differentiable complex vector bundle $E \to V$ is called foliated and analytic along the leaves if it has a local trivialisation with foliated transition functions wich are also analytic along the leaves, i.e. E induces local constant bundles, analytical along the leaves of $\mathcal{F}$.

We denote by : $\Gamma(U,E)$ the space of differentiable sections of E over the open set $U \subset V$, T , T^* the tangent and cotangent bundles of V, and $\mathcal{A}^{p_1p_2q_1q_2}(V)$ the sheaf of germs of differentiable forms on V of type (p_1,p_2,q_1,q_2),i.e. there local expresion is of the form:

$$\omega = \frac{1}{p_1!p_2!\,q_1!q_2!}\varphi_{\underline{a}_1\ldots a_{p_1}\,b_1\ldots b_{p_2}\cdot u_1\ldots u_{q_1}\bar{v}_1\ldots\bar{v}_{q_2}}\, dz^{\underline{a}}\wedge \ldots\wedge dz^{a_{p1}}\wedge dz^{b_1}\wedge\ldots\wedge dz^{b_{p2}}\wedge d\bar{z}^{u_1}\wedge\ldots\wedge d\bar{z}^{u_{q_1}}\wedge d\bar{z}^{v_1}\wedge\ldots\wedge d\bar{z}^{v_{q_2}}$$

We also recall that the complex differential operator $d = \partial + \bar{\partial}$ has relative to the forms of type (p_1,p_2,q_1,q_2) the following decomposition

$$d = d_{1ooo} + d_{o1oo} + d_{oo1o} + d_{2o-1o} + d_{11-1o} + d_{o11-1} + d_{1o-11} + d_{o2o-1} + d_{11o-1}$$

where the indices define the type of the corresponding operator; a form ω analytical along the leaves , i.e. $d_{ooo1}\omega = 0$ is of

type $(p_1, p_2, q_1, 0)$.

Given a $(V, \mathcal{F})$ structure and a complex (differentiable) vector bundle $E \to V$ we have:

Definition 2. A $d_{0001} = \hat{d}$- connection on $E \to V$ is a differential operator

$$D: \Gamma(E) \longrightarrow \Gamma(T^* \otimes E), \quad \Gamma(E) = \Gamma(V, E)$$

which is a $\hat{d}$-derivation, i.e.:

$$D(\varphi \xi) = \hat{d}\varphi \otimes \xi + \varphi D\xi ,$$

with $\xi \in \Gamma(E)$ and $\varphi \in \Gamma(T^* \otimes E)$.

Given a local frame $s = \{s_i\}$ for E, $s_i \in \Gamma(U, E)$ we recall that the connection matrix $\theta = (\theta_\beta^\alpha)$ is defined by

$$Ds_\alpha = \sum_\beta \theta_\alpha^\beta \otimes s_\beta , \quad \theta_\alpha^\beta \in \Gamma(U, T^*)$$

and passing to another frame $\tilde{s} = \{\tilde{s}_i\}$ - usir a corresponding matrix A of E over $\tilde{U} \subset V$, on $U \cap \tilde{U}$ we have:

$$\tilde{\theta} = A^{-1} \hat{d} A + A^{-1} \theta A$$

In the classical way we may associate to D the curvature matrix Θ relative to s which is a matrix of 2-forms

$$\Theta = \hat{d}\theta_\alpha^\beta + \sum \theta_\alpha^\sigma \wedge \theta_\sigma^\beta .$$

By a change of frame Θ verifies the relation

$$\tilde{\Theta} = A^{-1} \Theta A ;$$

it also verifies Bianchi's identity

$$\hat{d}\Theta = \Theta \wedge \theta - \theta \wedge \Theta = [\Theta, \theta] .$$

Proposition 1. A $\hat{d}$-connection may be obtained as the part of type $(0,0,0,1)$ of a d"-connection [1].

Proof: $d'' = d_{0100} + d_{0001} + d_{11-10} + d_{10-11} + d_{02 0-1} = \bar{\partial}$; for $X \in T$ we have $X = X^{1,0} + X^{0,1}$ with $X^{0,1} = X^{1000} + X^{0100} + X^{0010} + X^{0001}$; given a (local) section of E we deduce:

$$\nabla^{0,1} t(X) = \nabla t(X^{1000} + X^{0100} + X^{0010}) + Dt(X^{0001})$$

where $\nabla^{0,1}$ is the connection corresponding to d", and ∇ is a usual connection.

Using [2] we shall define now the Chern classes of $E \to V$.

Let $\mathcal{M}_r(\mathbb{C})$ be the set of r-matrices with complex elements, $\tilde{I}_k(\mathcal{M}_r)$ the $\mathbb{C}$-space of k-linear forms on $\mathcal{M}_r$ and $I_k(\mathcal{M}_r)$ the space of invariant homogeneous polynomials of degree k induced by $\tilde{I}_k(\mathcal{M}_r)$, i.e

$$\varphi_k : \mathcal{M}_r \to \mathbb{C}, \quad \varphi_k(gAg^{-1}) = \varphi_k(A) ,$$

with $g \in GL(r,\mathbb{C})$.

The determinant of an element of $\mathcal{M}_r(\mathbb{C})$ is a mapping

$$\det : \mathcal{M}_r \to \mathbb{C},$$

wich is evident an element of $I_r(\mathcal{M}_r)$ for wich:

$$\det(I+A) = \sum_{k=0}^{r} \varphi_k(A),\quad A \in \mathcal{M}_r,\ \varphi_k \in I_k(\mathcal{M}_r)$$

and I is the identity of $\mathcal{M}_r$.

We extend now the action of $\varphi_k \in I_k(\mathcal{M}_r)$ to a space wich will be specified later.

If U is open in V and $A_i \otimes w_i \in \mathcal{M}_r(U) \otimes_{\mathcal{A}(U)} \mathcal{A}^p(U)$, $p = p_1+p_2+\ldots$,

$\hat{d}\, w_i = 0$, we define

$$\varphi_U(A_1 \otimes w_1, \ldots, A_k \otimes w_k) = w_1 \wedge \ldots \wedge w_k\ \varphi_k(A_1, \ldots, A_k)$$

If $\xi_i \in \Gamma(U, \Lambda^p T^* \otimes \mathrm{Hom}(E,E))$, we define

$$\varphi_U(\xi_1, \ldots, \xi_k) = \varphi_U(\xi_1(s), \ldots, \xi_k(s))$$

and the definition does not depend of the choice of frame because

$$\varphi_U(\xi_1(s\tilde{s}), \ldots, \xi_k(s\tilde{s})) =$$

$$= \varphi_U(A^{-1}\xi_1(s)A, \ldots, A^{-1}\xi_k(s)A) = \varphi_U(\xi_1(s), \ldots, \xi_k(s))$$

by the invariance of φ and the induced invariance of φ when acting on matrix with differential form coefficients.

So we get an extension of φ_k to V by

$$\varphi_V : \Gamma(\Lambda^p T^* \otimes \mathrm{Hom}(E,E)) \underbrace{\times \ldots \times}_{k\text{-times}} \Gamma(\Lambda^p T^* \otimes \mathrm{Hom}(E,E)) \longrightarrow$$

$$\longrightarrow \Gamma(\Lambda^{p+k} T^* \otimes \mathrm{Hom}(E,E)).$$

wich restricted to the diagonal induces the action of $\varphi_k \in I_k(\mathcal{M}_r)$ on $\Gamma(\Lambda^p T^* \otimes \mathrm{Hom}(E,E))$ denoted by

$$\Phi_V : \Gamma(\Lambda^p T^* \otimes \mathrm{Hom}(E,E)) \to \Gamma(\Lambda^{p+k} T^* \otimes \mathrm{Hom}(E,E))$$

If $\varphi \in I_k(\mathcal{M}_r)$ then $\Phi_V(\Theta_E(D))$ where $\Theta_E(D)$ is the curvature of the considered $\hat{d}$-connection, is a 2k-global form on V and it could be proved in the classical way that $\Phi_V(\Theta_E(D))$ is independent of the choice of frame and that it defines a $\hat{d}$-closed global form of type(0,0,0,2k). The Dolbeault cohomology class of this form is independent of the choice of the $\hat{d}$-connection.

Definition 3. The k-$\hat{d}$-Chern form of E relative to D is the Dolbeault cohomology class represented by the $\hat{d}$-closed form

$$C_k(E,D) = \Phi_V\left(\left(\frac{i}{2\pi}\right)^k \Theta_E(D)\right) \in \Gamma(\Lambda^{2k} T^*)$$

and the $\hat{d}$-total Chern form of E relative to D is

$$c(E,D) = \sum_{k=0}^{r} c_k(E,D), \quad r = \text{rank } E.$$

If O_f is the sheaf of germs of foliate functions on V analytical along the leaves , then

$$c_k(E,D) \in H^{2k}(V, O_f)$$

Proposition 2. If $E \to V$ is foliated and analytic along the leaves then $c(E,D)=0$

Proof.: $E \to V$ is analytic along the leaves;then there is a connection on E such that its connection matrix is local of the type $(p_1,p_2,q_1,0)$, with $p_i + p_i + q_i = 1$; then its connection matrix corresponding to its part of type $(0,0,0,1)$vanishes and so its curvature matrix vanishes.

Then,we may conclude that the $\hat{d}$-Chern form of a complex vector bundle is an obstruction to the existence of foliate bundles analytic along the leaves wich are equivalent with the given one.

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CLASE CHERN DE VARIETATI FOLIATE

Rezumat

In această lucrare sînt construite formele Chern ale unui tip special de varietăţi complexe foliate, aşa numitele varietăţi foliate analitice de-a lungul foliilor.

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MORPHISMES SEECIALES EN CATEGORIES FLOUES ET IMMERSIONS (3)

D.Dăianu

Nous continuons la présentation des quelques morphismes spéciales des catégories floues, commencée dans [3] pour la catégorie Rel et continuée en [4] pour la catégorie Ens(L); nous marquons aussi les liaisons entre les catégories Ens, Rel, Set(L) $SEt_f(L)$, $Set_g(L)$, établies par des foncteurs fidéles.

4. Morphismes retractants, sectionnants et isomorphismes dans la catégories Ens(L)

4.1. Théorème. $R: A \longrightarrow B$ est un morphisme retractant si :

$$\bigvee_{b\in B_{a_0}\setminus \bigcup_{a\in A\setminus\{a_0\}} B_a} R(a_0,b) = 1,$$

pour chaque $a_0 \in A$.

Démonstration. Soit $R': B \longrightarrow A$, $(b,a_0) \in B \times A$ et

$$R'(b,a_0) = \begin{cases} R(a_0,b) & \text{si } b \in B_{a_0} \setminus \bigcup_{a\in A\setminus\{a_0\}} B_a \\ 0 & \text{en reste} \end{cases}$$

Alors $R'(a_0,a_0) = \bigvee_{b\in B}[R(a_0,b) \wedge R'(b,a_0)] = \bigvee_{b\in B_{a_0}\setminus \bigcup_{a\in A\setminus\{a_0\}} B_a} R(a_0,b) =$ 1; soit maintenant $a_1,a_2 \in A$, $a_1 \neq a_2$: $R'R(a_1,a_2) =$ $\bigvee_{b\in B}[R(a_1,b) \wedge R'(b,a_2)] = \bigvee_{b\in (B_{a_1}\setminus \bigcup_{a\in A\setminus\{a_1\}} B_a) \cap (B_{a_2}\setminus \bigcup_{a\in A\setminus\{a_2\}} B_a)} [R(a_1,b) \wedge R(a_2,b)] =$ $= 0$, donc $R'R = 1_A$ et R est retractant.

4.2. Théorème. $R: A \longrightarrow B$ est un morphisme sectionnant si :

$$\bigvee_{a\in A_{b_0}\setminus \bigcup_{b\in B\setminus\{b_0\}} A_b} R(a,b_0) = 1,$$

pour chaque $b_0 \in B$.

Démonstration. Soit $R': B \longrightarrow A$ définit pour chaque $(b_0,a) \in$

$\in B \times A$ et :

$$R'(b_o,a) = \begin{cases} R(a,b_o) & \text{si } a \in A_{b_o} \setminus \bigcup_{b \in B \setminus \{b_o\}} A_b \\ 0 & \text{en reste} \end{cases} ;$$

on vérifie l'égalité $RR' = 1_B$.

4.3. Conséquence. Si pour chaque $a_o \in A$ il existe $b_o \in B$ ainsi que $R(a_o,b_o) = 1$ et $R(a,b_o) = 0$ si $a \neq a_o$, alors le morphisme R est retractant.

4.4. Conséquence. Si pour chaque $b_o \in B$ il existe $a_o \in A$ ainsi que $R(a_o,b_o) = 1$ et $R(a_o,b) = 0$ si $b \neq b_o$, alors le morphisme R est sectionnant.

4.5. Conséquence. Si pour chaque $(a_o,b_o) \in A \times B$ il existe $(a_1,b_1) \in A \times B$ ainsi que $R(a_o,b_1) = 1 = R(a_1,b_o)$ et $R(a,b_1) = 0 = R(a_1,b)$ pour chaque $a \in A$ et $b \in B$ avec $a \neq a_1$ et $b \neq b_1$, alors R est isomorphisme.

4.6. Théorème. Si la latice L vérifit les conditions

(L_1) $a \wedge b = 0$ implique $a = 0$ où $b = 0$,

(L_2) $\bigvee_{i \in I} a_i = 1$ implique l'existence de $i_o \in I$ ainsi que $a_{i_o} = 1$,

alors Ens(L) est une catégorie équilibrée.

4.7. Exemple. Si L est une latice finie, complètement ordonée en raport avec l'ordre induit des ses opérations, alors L satisfait les conditions (L_1) et (L_2) et alors Ens (L) sera equilibrée.

5. Immersions.

5.1. Théorème. La catégorie Ens on immerge dans la catégorie Set(L) et dans la catégorie Ens(L) par des foncteurs covariants (contravariants) qui conservent les monomorphismes et les épimorphismes (transforment les monomorphismes en épimorphismes et les épimorphismes en monomorphismes).

Démonstration. Le foncteur $F : \text{Ens} \longrightarrow \text{Set}(L)$, $F(X) = (X, 1_X^L)$ pour chaque objet X de Ens, où $1_X^L : X \longrightarrow L$, $1_X^L(x) = 1$, pour chaque $x \in X$ et qui pour chaque application $f : X \longrightarrow Y$ attache $F(f)(x) = f(x)$, $F(f) : (X, 1_X^L) \longrightarrow (Y, 1_Y^L)$; $1_Y^L \circ F(f) = 1_X^L$ dans Set(L) et F est monofoncteur et fidèle. Le foncteur $F_1 : \text{Ens} \longrightarrow \text{Ens}(L)$:

- pour chaque $A \in \text{Ob Ens}$ $F_1(A) = A$
- pour chaque $f \in \text{Ens}(A,B)$, $F_1(f)(a,b) = \begin{cases} 1 & \text{pour } b = f(a) \\ 0 & \text{en reste} \end{cases}$

pour chaque $(a,b) \in A \times B$, est correctement définit parce que , évidement $F_1(f) \in \text{Ens}(A,B)$ et $F_1(1_A) = 1_{F_1(A)}$; soit $f \in \text{Ens}(A,B)$, et $g \in \text{Ens}(B,C)$:

$(F_1(gof))\ (a,c) = \begin{cases} 1 & \text{pour } (gof)(a) = c \\ 0 & \text{en reste} \end{cases}$

$(F_1(g) \circ F(f))\ (a,c) = \bigvee_{b \in B} (F_1(f)(a,b) \wedge F_1(g)(b,c)) =$

$= \begin{cases} 1 \text{ si } b = f(a) \text{ et } c = g(b) \\ 0 \text{ en reste} \end{cases}$, d'où $F_1(g \circ f) = F_1(g) \circ F_1(f)$

et donc F_1 est un monofoncteur fidèle. Ens est une catégorie équilibrée et donc les images des injections sont des morphismes rétractants et ceux des surjections sont des morphismes séctionant.

Le foncteur F_2: Ens $\longrightarrow$ Ens(L) définit :

$(F_2(f))\ (b,a) = \begin{cases} 1 & \text{si } b = f(a) \\ 0 & \text{en reste} \end{cases}$

pour chaque $(b,a) \in B \times A$ est contrevariant et Vérifie les affirmations de la théorème.

5.2. Théorème. La catégorie Set(L) on immerge dans $Set_f(L)$; la catégorie $Set_f(L)$ est sous-catégorie de la catégorie $Set_g(L)$; la catégorie $Set_g(L)$ est sous-catégorie de la catégorie Ens (L).

Démonstration. Soit F: Set(L) $\longrightarrow$ $Set_f(L)$ le foncteur définit sur les objets par $F(X,f) = (X,F)$ et sur les morphismes par :

$(F(\alpha))(x,y) = \begin{cases} f(x) & \text{si } y = (x) \\ 0 & \text{en reste} \end{cases}$

où $\alpha : (X,f) \longrightarrow (Y,g)$ est un morphisme en Set(L). Soit α et β deux morphismes composables: $\alpha \in Set(L)[(X,f),(Y,g)]$ et $\beta \in Set(L)[(Y,g),(Z,h)]$

Alors $F(\beta \circ \alpha)\ (x,y) = \begin{cases} f(x) & \text{si } \alpha(x) = z \\ 0 & \text{en reste} \end{cases}$ et $[F(\beta) \circ F(\alpha)]\ (x,z)$

$= \bigvee_{y \in B} [F(\alpha)(x,y) \wedge F(\beta)(y,z)] = \begin{cases} f(x) \wedge g(y) & \text{si } y = \alpha(x),\ z = \beta(y) \\ 0 & \text{en reste} \end{cases}$

donc $F(\beta \circ \alpha) = F(\beta) \circ F(\alpha)$; $F(\alpha) \in$ Hom $Set_f(L)$ parce que pour chaque $x \in X$ et $y \in Y$ $(F(\alpha))(x,y) \leq f(x) \leq (g\alpha)(x) = f(x) \wedge g(y)$. Evidemment F est un foncteur fidèle et monofoncteur. Soit R: $(X,f) \longrightarrow (Y,g)$ une relation floue ainsi que $R(x,y) \leq f(x) \wedge g(y)$; alors : $\bigvee_{x \in X} [R(x,y) \wedge f(x)] \leq g(y)$, donc $R \in$ Hom $Set_g(L)$. La dernière affirmation est évidemmente (voir aussi [8]).

5.3. Théorème. Soit L un latice distributive, avec le premier et le dernier élément, complète en rapport de la réunion; alors Ens(L) est une catégorie ordonnée, σ -complète.

Démonstration. En ce qui concerne les catégories ordonnée voir [1] où [5]. On muni chaque Ens(L)[A,B] avec l'ordre naturel :

$R_1 \leq R_2 \Longleftrightarrow R_1(a,b) \leq R_2(a,b)$ pour chaque $(a,b) \in A \times B$

Nous considérons les morphismes suivantes :

$$D \xrightarrow{R'} A \underset{R_2}{\overset{R_1}{\rightrightarrows}} B \xrightarrow{R''} C$$

et soit $R_1 \leq R_2$; parce que : $R_1R'(d,b) = \bigvee_{a \in A} [R'(d,a) \wedge R_1(a,b)] \leq$

$\bigvee_{a \in A} [R'(d,a) \wedge R_2(a,b)] = R_2R'(d,b)$, pour chaque $(d,b) \in D \times B$ il

suit : $R_1R' \leq R_2R'$ et de même $R''R_1 \leq R''R_2$. Soit maintenant une famille des morphismes $R_i \in Ens(L)[A,B]$, $i \in I$; on definit le morphisme $\underset{i\in I}{U} R_i \in Ens(L)[A,B]$,par :

$$(\underset{i\in I}{U} R_i)(a,b) = \underset{i\in I}{V} R_i(a,b), \text{ pour chaque } (a,b) \in A \times B ;$$

évidement c'est la réunion de la famille.

$$R(\underset{i\in I}{U} R_i)(a,c) = \underset{i\in I}{V}[(\underset{i\in I}{U} R_i)(a,b) \wedge R(b,c)] = \underset{b\in B}{V}[\underset{i\in I}{V} R_i(a,b) \wedge R(b,c)] =$$

$$\underset{b\in B}{V}\underset{i\in I}{V}[R_i(a,b) \wedge R(b,c)] = \underset{i\in I}{V}\underset{b\in B}{V}[R_i(a,b) \wedge R(b,c)] = \underset{i\in I}{V} RR_i(a,c)$$

$$= \underset{i\in I}{U}(RR_i)(a,c), \text{ pour chaque } (a,c) \in A \times C \text{ et aussi } ((\underset{i\in I}{U} R_i)R)(c,b) =$$

$$= \underset{a\in A}{V}[(\underset{i\in I}{U} R_i)(a,b) \wedge R(c,a)] = \underset{a\in A}{V}\underset{i\in I}{V}[R_i(a,b) \wedge R(c,a)] =$$

$$= \underset{i\in I}{V}(R_iR)(c,b) = (\underset{i\in I}{U} R_iR)(c,b), \text{ pour chaque } (c,b) \in C \times B \text{ , où R est}$$

est dans le premier case un morphisme de $Ens(L)[B,C]$ et dans le deuxième cas un morphisme de $Ens(L)[C,A]$ et par conséquence :

$$R(\underset{i\in I}{U} R_i) = \underset{i\in I}{U}(RR_i) \quad \text{et} \quad (\underset{i\in I}{U} R_i)R = \underset{i\in I}{U}(R_iR)$$

et donc Ens(L) est une catégorie ordonnée, σ -complète.

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MORFISME SPECIALE IN CATEGORII FUZZY SI SCUFUNDARI (3)

Rezumat

După ce în [3] am caracterizat unele morfisme speciale în categoria relaţiilor binare Rel, am continuat în [4] si în prezenta lucrare cu asemenea caracterizări în categoria relaţiilor fuzzy Ens(L) Am stabilit totodată si cîţiva monofunctori fideli care realizează scufundările $Ens \hookrightarrow Set(L) \hookrightarrow Set_f(L) \hookrightarrow Set_\sigma(L) \hookrightarrow Ens(L)$ si $Rel \hookrightarrow Ens(L)$. In final am arătat că Ens(L) este o categorie σ-completă .

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ON THE QUASI CONTINUITY OF THE LIMITS FOR NETS OF MULTIFUNCTIONS

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1. Introduction.In 1971 R.E.Smithson ([8]) gives sufficient conditions for the continuity of limit multifunctions for a net of continuous multifunctions defined on a topological space into a uniform space.

In [1] , [2] , [3] sufficient conditions for the transfer of the continuity from the terms of multifunctions net to the limit are given in the case in which the ranges are to be found in uniform space.

If we think the range multifunctions into topological space some difficulties appear because in this case the notion of convergence must be defined.

In [4] sufficient conditions for the upper semicontinuity of upper topological limit for a multifunction net are given.

In this note sufficient conditions for the quasi continuity of multifunction limit for a net of quasi continuous multifunctions are given.

2. Preliminary notions. In the followings we shall note by $(X,\mathcal{T}_X)$ and $(Y,\mathcal{T}_Y)$ topological spaces, by $(Y,\mathcal{U})$ an uniform space, and by $(D,\geqslant)$ a directed set. (F_α) will be a net of multifunctions, $F_\alpha : X \longrightarrow Y, \alpha \in D$, and F will be a multifunction on X into Y.

Definition 2.1. 1) (F_α) converges upper pointwise to F on X if for every $x\in X$ and for every open set G which contains F(x), there is $\beta(x,G)\in D$ such that $F_\alpha(x) \subset G$, for all $\alpha \geqslant \beta(x,G)$.

2) (F_α) converges lower pointwise to F on X if for every $x\in X$ and for every set G which intersects F(x), there is $\beta(x,G)\in D$ such that $F_\alpha(x)\cap G \neq \emptyset$, for all $\alpha \geqslant \beta(x,G)$.

3) (F_α) converges pointwise to F on X if it converges upper pointwise and lower pointwise to F.

Definition 2.2. 1) (F_α) converges quasi upper r-uniformly (q.u.r.u.)to F on X if:

(i) (F_α) converges pointwise to F on X

(ii) for every open set G with $F^{-1}(G) \neq \emptyset$ and for every $\beta \in D$, there is $\alpha \gg \beta$ such that $F_\alpha(x) \subset G$, for all $x \in F^{-1}(G)$.

2) (F_α) converges quasi lower r-uniformly (q.l.r.u.) to F on X if:

(i) (F_α) converges pointwise to F on X

(ii)' for every open set G with $F_{-1}(G) \neq \emptyset$ and for every $\beta \in D$, there is $\alpha \gg \beta$ such that $F_\alpha(x) \cap G \neq \emptyset$, for all $x \in F_{-1}(G)$.

3) (F_α) converges quasi r-uniformly (q.r.u.) to F on X if it converges q.u.r.u. and q.l.r.u. to F.

3. In this section we shall prove theorems which give sufficient conditions for the quasi continuity of limit for a net made of quasi continuous multifunctions.

Theorem 3.1. Let (F_α) be quasi uniformly convergent to F on X. If every F_α is quasi continuous and if F is point compact then F is quasi continuous.

Proof. Let $x_0 \in X$ be a point for which F_α are quasi continuous, $H \in \mathfrak{U}$, and $H' \in \mathfrak{U}$ a symmetric entourage such that $H'oH'oH' \subset H$. From the definition of quasi uniformly convergence (see [2]) we have

$$F_{h(\beta)}(x) \subset H'(F(x)),\ F(x) \subset H'(F_{h(\beta)}(x)) \quad (\forall)\ x \in G_\beta \qquad (1)$$

Let G_{β_0} be the open set which contains x_0 and U a neighborhood of x_0. From the quasi continuity of $F_{h(\beta_0)}$ in x_0, using theorem from [5], it follows that there is an open nonvoid set $V \subset U \cap G_{\beta_0}$ such that

$$F_{h(\beta_c)}(x) \subset H'(F_{h(\beta_0)}(x_0)),\ F_{h(\beta_c)}(x_0) \subset H'(F_{h(\beta_0)}(x)) \quad (\forall)\ x \in V \qquad (2)$$

From (1) we obtain

$$F_{h(\beta_0)}(x_0) \subset H'(F(x_0)),\ F(x_0) \subset H'(F_{h(\beta_c)}(x_0)) \qquad (1')$$

and

$$F_{h(\beta_c)}(x) \subset H'(F(x)),\ F(x) \subset H'(F_{h(\beta_c)}(x)) \quad (\forall) x \in V \qquad (1'')$$

From (1'') , (2) and (1') it follows that $F(x) \subset (H'oH'oH')(F(x_0))$ for all $x \in V$, hence

$$F(x) \subset H(F(x_0)) \quad (\forall)\ x \in V \qquad (3)$$

From (1'), (2) and (1'') it follows that $F(x_0) \subset (H'oH'oH')(F(x))$ for all $x \in V$, hence

$$F(x_0) \subset H(F(x)) \quad (\forall)\ x \in V \qquad (4)$$

From (3) and (4) , using theorem from [5] , it follows that F is quasi continuous in x_0.

Theorem 3.2. Let (F_α) be a net which converges q.l.r.u. to F, and F be point compact. If the topological space $(Y, \mathcal{T}_Y)$ is regular and every F_α is quasi u.s.c. ([7]), then F is quasi u.s.c.

Proof. We suppose that F is not quasi u.s.c. in $x_0 \in X$, but all F_α are quasi u.s.c. in x_0. Then there is an open set G which contains $F(x_0)$ and there is a neighborhood U of x_0 such that, for every open nonvoid set $V \subset U$, there is $x_V \in V$ with $F(x_V) \not\subset G$.

But $F(x_0) \cap (Y-G) = \emptyset$, $F(x_0)$ is compact, Y-G is closed and $(Y, \mathcal{T}_Y)$ is regular, so it follows that there are two disjoint open sets G_1, G_2 such that $F(x_0) \subset G_1$, $Y - G \subset G_2$.

From the pointwise convergence of (F_α) to F it follows that there is $\alpha_0 \in D$ such that

$$F_\alpha(x_0) \subset G_1 \qquad (\forall)\ \alpha \geq \alpha_0 \qquad (5)$$

But $F_{-1}(G_2) \neq \emptyset$ $(x_V \in F_{-1}(G_2))$ and (F_α) converges q.l.r.u. to F, so it follows that there is $\alpha_1 \geq \alpha_0$ such that $F_{\alpha_1}(x) \cap G_2 \neq \emptyset$, for all $x \in F_{-1}(G_2)$, hence $F_{\alpha_1}(x_V) \cap G_2 \neq \emptyset$, that means

$$F_{\alpha_1}(x_V) \not\subset G_1 \qquad (6)$$

Using (5) and (6) it follows that if F is not quasi u.s.c. in x_0, then F_{α_1} is not quasi u.s.c. in x_0, which contradicts the hypothesis.

Theorem 3.3. Let (F_α) be a net which converges q.u.r.u. to F on X. If the topological space $(Y, \mathcal{T}_Y)$ is regular and every F_α is quasi l.s.c. ([7]), then F is quasi l.s.c.

Proof. We suppose that F is not quasi l.s.c. in $x_0 \in X$, but all F_α are quasi l.s.c. in x_0. Then there is an open set G which intersects $F(x_0)$ and there is a neighborhood U of x_0 such that for every nonvoid open set $V \subset U$ there is $x_V \in V$ with $F(x_V) \cap G = \emptyset$.

Let $y_0 \in F(x_0) \cap G$ be arbitrary. Then $y_0 \notin (Y-G)$ and using the hypothesis that $(Y, \mathcal{T}_Y)$ is regular it follows that there are two nonvoid disjoint open sets G_1, G_2 such that $y_0 \in G_1$, $Y - G \subset G_2$. Hence $F(x_0) \cap G_1 \neq \emptyset$. Using the poinwise convergence of (F_α) to F it follows that there is $\alpha_0 \in D$ such that

$$F_\alpha(x_0) \cap G_1 \neq \emptyset \qquad (\forall).\ \alpha \geq \alpha_0 \qquad (7)$$

But $F^{-1}(G_2) \neq \emptyset$ $(x_V \in F^{-1}(G_2))$ and (F_α) converges q.u.r.u. to F, so it follows that there is $\alpha_1 \geq \alpha_0$ such that

$F_{\alpha_1}(x) \subset G_2$, for all $x \in F^{-1}(G_2)$, hence $F_{\alpha_1}(x_V) \subset G_2$, that implies

$$F_{\alpha_1}(x_V) \cap G_1 = \emptyset \qquad (8)$$

Using (7) and (8) it follows that if F is not quasi l.s.c. in x_0, then there is α_1 such that F_{α_1} is not quasi l.s.c. in x_0, which contradicts the hypothesis.

From theorems 3.2. and 3.3. it follows

Theorem 3.4. Let (F_α) be a net which converges q.r.u. to F which is point compact on X .If the topological space $(Y, \mathcal{T}_Y)$ is regular and every F_α is quasi continuous then F is quasi continuous.

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ASUPRA QUASI CONTINUITATII LIMITELOR SIRURILOR GENERALIZATE DE MULTIFUNCTII

Rezumat

In lucrare se dau condiţii suficiente pentru quasi continuitatea multifuncţiei limită a unui şir generalizat de multifuncţii quasi continue.

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ON OPERATOR IDEALS ON BANACH COUPLES

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Definition 1.1. By a Banach couple $\bar{E}=\{E_1,E_2\}$ we mean two Banach spaces E_j $(j=1,2)$ continuously included in a Hausdorff topological vector space.

To every Banach couple $\bar{E}$ one associate the Banach space $E_\Delta = E_1 \cap E_2$ and $E_\Sigma = E_1 + E_2$, with the norms:

$$\|x\|_{E_\Delta} := \max\{\|x\|_{E_1}, \|x\|_{E_2}\} \quad (x \in E_\Delta)$$

$$\|x\|_{E_\Sigma} := \inf\{\|x_1\|_{E_1} + \|x_2\|_{E_2} : x = x_1 + x_2,\ x_j \in E_j\} \quad (x \in E_\Sigma)$$

The pears of Banach couples form the interpolation segments. We denote by $[\bar{E},\bar{F}] = [E_1, E_2; F_1, F_2]$ the interpolation segment given by the couples $\{E_1, E_2\}$ and $\{F_1, F_2\}$. The class of operators acting between Banach couples is denoted by $\mathcal{L}$. In this case the components will be evidentiated for pears of Banach couples, i.e. for interpolation segments. For the segment $[\bar{E},\bar{F}]$ the components $\mathcal{L}[\bar{E},\bar{F}]$ mean the linear operators acting from E_Σ to F_Σ such that their restriction to E_j $(j=1,2)$ applig continuously E_j in F_j $(j=1,2)$

By analogy to the operator ideals on Banach spaces we define operator ideals with components on pears of couples

Definition 1.2. Given two operator ideals $\mathcal{A}_1$ and $\mathcal{A}_2$ on Banach spaces for an arbitrary interpolation segment $[E_1, E_2; F_1, F_2]$ we define $(\mathcal{A}_1 \cap \mathcal{A}_2)[E_1, E_2; F_1, F_2]$ to be the set of operators T from $\mathcal{L}[E_1, E_2; F_1, F_2]$ for which $T\dot{J}_{E_K} \in \mathcal{A}_K(E_K, F_K)$, $k=1,2$, where $\dot{J}_{E_K}$ is the canonical embedding of E_K into $E_1 + E_2$.

It can be proved that $\mathcal{A}_1 \cap \mathcal{A}_2$ is the operator ideal on Banach couples (see. [1])

Our goal is to extend the clos and rad procedure given in [2] for operator ideals on Banach spaces to the operator ideals on Banach couples. Applying this procedures to ideals on spaces we will obtain new ideals.

2. Closed operator ideals on Banach couples.

Let $\mathcal{A}$ be an operator ideal on Banach couples and S be an operator in $\mathcal{L}[E_1, E_2; F_1, F_2]$.

Definition 2.1. The operator S belongs to the closure $\mathcal{A}^{clos}$ if there exists a sequence of operators $S_1, S_2, \dots, S_n, \dots \in \mathcal{A}[E_1, E_2; F_1, F_2]$ such that $\lim_n \|S - S_n\|_{\mathcal{L}} = 0$.

Theorem 2.1. $\mathcal{A}^{clos}$ is an operator ideal on Banach couples.

Proof. We will verify the conditions in the definiton of an operator ideal on Banach couples (sec[1]).

(OIC.0) $\mathcal{A}$ being an operator ideal on Banach couples it results that $I_K \in \mathcal{A}[K,K;K,K]$, what implies the existence of a constant sequence $S_K = I_K$ such that $\lim_n \|I_K - S_n\|_{\mathcal{L}} = 0$. Hence $I_K \in \mathcal{A}^{clos}[K,K;K,K]$.

(OIC.1). For every $T_1, T_2 \in \mathcal{A}^{clos}[E_1, E_2; F_1, F_2]$ we show that $T_1 + T_2 \in \mathcal{A}^{clos}[E_1, E_2; F_1, F_2]$. From $T_1 \in \mathcal{A}^{clos}[E_1, E_2; F_1, F_2]$ it results that there is a sequence $S_1, S_2, \dots \in \mathcal{A}[E,F]$ such that $\lim_n \|T_1 - S_n\|_{\mathcal{L}} = 0$. Analogously, for T_2. Then $\lim_n \|(T_1 + T_2) - (S_n + S_n')\|_{\mathcal{L}} = 0$, on hence $T_1 + T_2 \in \mathcal{A}^{clos}[E,F]$.

(OIC.2) Let $T \in \mathcal{L}[E,X]$, $S \in \mathcal{A}^{clos}[X,Y]$ and $R \in \mathcal{L}[Y,F]$. Il follows that there exists the sequence $S_1, S_2, \dots \in \mathcal{A}[X,Y]$ such that $\lim_n \|S - S_n\|_{\mathcal{L}} = 0$ and $\lim_n \|RST - RS_nT\|_{\mathcal{L}} = \lim_n \|RT(S - S_n)\|_{\mathcal{L}} \le \|R\|_{\mathcal{L}} \cdot \|T\|_{\mathcal{L}} \cdot \lim_n \|S - S_n\|_{\mathcal{L}} = 0$. Hence $RST \in \mathcal{A}^{clos}[E,F]$.

Theorem 2.2. The rule $clos: \mathcal{A} \to \mathcal{A}^{clos}$ is a hull procedure.

Proof. We must show that it is a monotone, idempotent procedure and $\mathcal{A} \subseteq \mathcal{A}^{clos}$.

(M) Let $\mathcal{A} \subseteq \mathcal{B}$ be operator ideals on Banach couples and let be the operator $S \in \mathcal{L}[E,F]$, $S \in \mathcal{A}^{clos}[E,F]$. Than there is a sequence $S_1, S_2, \dots \in \mathcal{A}[E,F]$ such that $\lim_n \|S - S_n\|_{\mathcal{L}} = 0$. From $\mathcal{A} \subseteq \mathcal{B}$ it follows that (S_n) is an operator sequence of $\mathcal{B}$ and hence $S \in \mathcal{B}^{clos}$, i.e $\mathcal{A}^{clos} \subseteq \mathcal{B}^{clos}$.

(I) We show that $\left(\mathcal{A}^{clos}\right)^{clos} = \mathcal{A}^{clos}$.

Let $S \in \left(\mathcal{A}^{clos}\right)^{clos}[E,F]$. Then there exists a sequence of operators $S_1, S_2, \dots \in \mathcal{A}^{clos}[E,F]$ such that $\lim_n \|S - S_n\|_{\mathcal{L}} = 0$. Consider the operator $S_n \in \mathcal{A}^{clos}[E,F]$ for which $\|S - S_n\|_{\mathcal{L}} < \frac{1}{2n}$. There exists $T_n \in \mathcal{A}[E,F]$ such that $\|S_n - T_n\|_{\mathcal{L}} < \frac{1}{2n}$. We have $\|S - T_n\|_{\mathcal{L}} = \|S - S_n + S_n - T_n\|_{\mathcal{L}} \le \|S - S_n\|_{\mathcal{L}} + \|S_n - T_n\|_{\mathcal{L}} < \frac{1}{n}$ and by limit when $n \to \infty$ it follows that $S \in \mathcal{A}^{clos}[E,F]$.

Finally, let as show that $\mathcal{A} \subseteq \mathcal{A}^{clos}$. Let $S \in \mathcal{A}[E,F]$. That it exists a constant sequence $S_n = S, \forall\, n \in N$, $S_n \in \mathcal{A}[E,F]$ such that $\lim_n \|S - S_n\|_{\mathcal{L}} = 0$, i.e. $S \in \mathcal{A}^{clos}[E,F]$.

Proposition 2.1. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be operator ideals on Banach spaces .We have:

$$(\mathcal{A}_1 \wedge \mathcal{A}_2)^{clos} \subseteq \mathcal{A}_1^{clos} \wedge \mathcal{A}_2^{clos}.$$

Proof. Let $T \in \mathcal{L}[\bar{E},\bar{F}]$, $T \in (\mathcal{A}_1 \wedge \mathcal{A}_2)^{clos}[\bar{E},\bar{F}]$. It exists a sequence $S_1, S_2, \ldots \in (\mathcal{A}_1 \wedge \mathcal{A}_2)[\bar{E},\bar{F}]$ such that $\lim_n \| T - S_n \|_{\mathcal{L}} = 0$. Applying def.1.2 it follows that $S_n J_{E_1} \in \mathcal{A}_1(E_1,F_1)$ and $S_n J_{E_2} \in \mathcal{A}_2(E_2,F_2)$, $\forall n \in N$ .Hence there exists a sequence of operators $S_1 J_{E_1}, S_2 J_{E_1}, \ldots \in \mathcal{A}_1(E_1,F_1)$ such that $\lim_n \| TJ_{E_1} - S_n J_{E_1} \| = \lim_n \| (T - S_n) J_{E_1} \| = 0$, what implies that $TJ_{E_1} \in \mathcal{A}_1^{clos}(E_1,F_1)$. Analogously can be showed that $TJ_{E_2} \in \mathcal{A}_2^{clos}(E_2,F_2)$ .It results that $T \in (\mathcal{A}_1^{clos} \wedge \mathcal{A}_2^{clos})[\bar{E},\bar{F}]$.

Definition 2.2. An operator ideal on Banach couples to is called closed if $\mathcal{A} = \mathcal{A}^{clos}$. If $\mathcal{A}_1$ and $\mathcal{A}_2$ a closed ideals than the inequality in Proposition 2.2.becomes an equality.It is known that the operator ideals on Banach spaces $\mathcal{S}$(approximable), $\mathcal{R}$(compact)and $\mathcal{W}$(weakly compact)are closed.Then $\Sigma = \mathcal{S} \wedge \mathcal{S}$, $\mathcal{W} = \mathcal{W} \wedge \mathcal{W}$, $\mathcal{R} = \mathcal{R} \wedge \mathcal{R}$ are closed operator ideals on Banach couples .Applying the clos procedure we get $(\mathcal{R} \wedge \mathcal{S})^{clos} = \mathcal{R} \wedge \mathcal{S}$, $(\mathcal{L} \wedge \mathcal{W})^{clos} = \mathcal{L} \wedge \mathcal{W}$

3. The radical of an operator ideal on Banach couples.

Let $\mathcal{A}$ be an operator ideal on Banach couples.An operator $S \in \mathcal{L}[\bar{E},\bar{F}]$ belongs to the radical $\mathcal{A}^{rad}$ if for every operator $L \in \mathcal{L}[\bar{F},\bar{E}]$ there exist the operators $U \in \mathcal{L}[\bar{E},\bar{E}]$ and $X \in \mathcal{A}[\bar{E},\bar{E}]$ such that $U(I_\Sigma - LS) = I_\Sigma - X$.

Theorem 3.1. $\mathcal{A}^{rad}$ is an operator ideal on Banach couples.

Proposition 3.1.The rule $rad: \mathcal{A} \to \mathcal{A}^{rad}$ is a hull procedure.

Proposition 3.2.For every operator ideal $\mathcal{A}$ on Banach couples, the ideal $\mathcal{A}^{rad}$ is closed. It can be proved that $(\mathcal{A}^{rad})^{clos} = \mathcal{A}^{rad}$.

Proposition 3.3. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be operator ideals on Banach spaces. Then $(\mathcal{A}_1 \wedge \mathcal{A}_2)^{rad} \subseteq \mathcal{A}_1^{rad} \wedge \mathcal{A}_2^{rad}$

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ASUPRA UNOR IDEALE DE OPERATORI PE CUBLURI BANACH

Rezumat

In lucrare se definesc procedurile clos şi rad pentru ideale de operatori pe cupluri Banach şi se studiază proprietăţile idealelor obţinute cu ajutorul acestor proceduri.

Seminarul de Matematică şi Fizică
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FUNKTIONEN AUF NUMEROIDALEN STRUKTUREN

Matei St.

In [1] und [2] wurden die Axiome, Eigenschaften und Folgen einer numeroidalen Struktur behandelt.

Definition 1. Gegeben der Numeroid $\mathcal{N}$. Man bezeichnet als numeroidale Funktion von n Variabeln die Abbildung $f: \mathcal{N}^n \to \mathcal{N}$

Folge 1. Die Menge aller numeroidalen Funktionen von n Variabeln:

(1) $\{f[X] = \{f(x_1, x_2, \ldots, x_n\}$ bildet ein Numeroid.

Man bezeichnet mit $\mathcal{N}_1$ die Menge aller Funktionen von N zu $\mathcal{N}$:

(2) $\mathcal{N}_1 = \{f / f: N \to \mathcal{N}\}$

Folge 2. Die Menge $\{\mathcal{N}_1, +, .\}$ ist ein Numeroid

Satz 1. Gegeben die Abbildung $\varphi: \mathcal{N} \to \mathcal{N}_1$ für welche:

(3) $\varphi(a+b) = \varphi(a) + \varphi(b)$; $\varphi(ab) = \varphi(a)\varphi(b)$; $\varphi(\ddot{a}) = \varphi(b) \Rightarrow a = b$, $\forall a, b \in \mathcal{N}$

Definition 2. Die Abbildung $\varphi: \mathcal{N} \to \mathcal{N}_1$ welche (3) erfüllt, heist injektiver Homomorfismus von Numeroiden

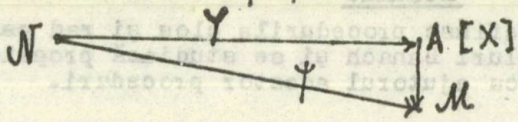
Definition 3. Der Numeroid $\{\mathcal{N}_1, +, ., \varphi\}$ heist Struktur von $\mathcal{N}$-Numeroid mit strukturellen Homomorfismus

Satz 2. Gegeben, A [X] Numeroid Polynome mit einer Unbekannte mit Koefizienten aus dem Numeroiden $\mathcal{N}$, $\mathcal{M}$ ein $\mathcal{N}$-Numeroid und $x \in \mathcal{N}$. Dann existiert ein einziger Hpmomorfismus von $\mathcal{N}$-Numeroiden u: A [X] $\to \mathcal{M}$; so das u(X) =x.

Beweis. Gegeben $\psi: \mathcal{N} \to \mathcal{M}$ ein struktur Homomorfismus des $\mathcal{N}$-Numeroiden . Man definiert die Abbildung u folgender Weise wenn $A \in A[X]$, $A = \sum_{i=0}^{m} a_i X^i$, dann $\mu(A) = \sum_{i=0}^{m} a_i x^i$

Die Abbildung u ist ein Homomorfismus von $\mathcal{N}$-Numeroiden wenn gezeigt wird dass: $u\varphi = \psi$, dass heißt , dass folgende Di .gramm:

(4)



komutativ ist. Für jedwelches $a \in \mathcal{N}$ gilt:

(5) $(u\varphi)(a) = u[\varphi(a)] = \mu(a) = \psi(a)\, X^{\circ} = \psi(a) x^{\circ} = \psi(a) \Rightarrow u\varphi = \psi$;

Es sei $u' : A[X]$ ein anderer Homomorfismus von $\mathcal{N}$-Numeroidem mit deselben Eigenschaften wie u dass heißt $u'\varphi = \psi$ und $u'(X) = x$ dann gilt:

(6) $u'(A) = \sum_{i=o}^{m} u'(a) \left[u'(X)\right]^{i} = \sum_{i=o}^{m} a_i x^{i} = u(A), \forall A \in A[X]$

also $u' = u$ und der Satz ist bewiesen wurden.

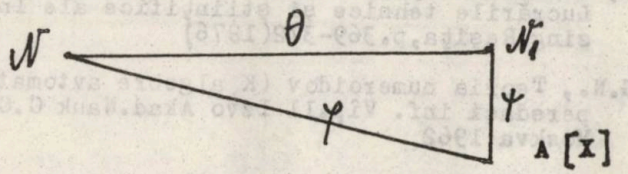
Satz 3. Ein $\mathcal{N}$-Numeroid ist isomorf mit dem $\mathcal{N}$- Numeroiden der Polynome mit einer Unbekannten mit Koefizienten in $\mathcal{N}$, dann und nur dann wenn ein Element $x \in \mathcal{N}_1$ existiert so das $: A[X] = \mathcal{N}_1$ und $\mathcal{N}_1$ hat die Eigenschaft : jedwelche Abbildung $\varphi : \mathcal{N} \to \mathcal{M}$ $\mathcal{N}$ ein Numeroid und $y \in \mathcal{M}$ existiert ein einzigler Homomorfismus mit $\mathcal{N}$- Numeroiden $\psi : \mathcal{N}_1 \to \mathcal{M}$ mit $\psi(x) = y$

Beweis. Wenn $\mathcal{N}_1$ ist ein Numeroid der Polynome mit einer Unbekannten, dann laut Satz 2 folgt das $\mathcal{N}_1$,

Eigenschaft aus der Voraussetzungbesitz, $\mathcal{N}_1 = A[x]$ und für jedwelches $\mathcal{N}$-Numeroid $\mathcal{M}$ und $y \in \mathcal{M}$ existiert ein einsiger Homomorfismus von $\mathcal{N}$-Numeroiden $u : \mathcal{N}_1 \to \mathcal{M}$, so das $u(x) = y$

Resiproke. Wen $\mathcal{N}_1$ hat die Eigenschaft aus der Voraussetzung erfillt, dann existiert ein einziger Homomorfismus $\psi : \mathcal{N}_1 \to A[X]$ mit $\psi(x) = X$ so dass folgendes Diagramm:

(7)



θ - struktureller Homomorfismus des $\mathcal{N}$- Numeroiden $\mathcal{N}_1$ kommutativ ist, dass heißt $\psi \circ \theta = \varphi$

Es sei $\alpha \in \mathcal{N}_1$ für welches $\varphi(\alpha) = 0$

Weil $\mathcal{N}_1 = A[X]$ folgt

(8) $\alpha = \sum_{i=a}^{m} a_i x^{i}$, $\psi(\alpha) = \sum_{i=o}^{m} \psi(a_i)\left[\psi(x)\right]^{i} = \sum_{i=o}^{m} a_i X_{=o}^{i} \Rightarrow a_i = 0$

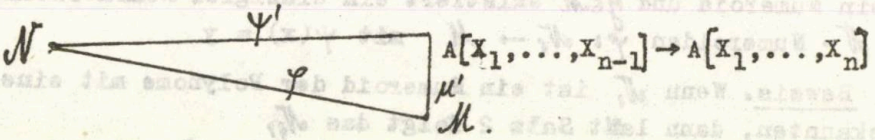
dass heißt $\alpha = 0$.

Folgt $\ker\varphi = 0$ und φ ist injektiv. Es sei $A \in A[X]$, $A = \sum_{i=0}^{m} b_i X^i$ dann $\beta = \sum_{i=0}^{m} b_i x^i$ ist ein Element aus $\mathcal{N}_1$ und $\varphi(\beta) = A$. Also φ ist surjektiv. Der Homomorfismus φ ist bijektiv.

Satz 4. Es sei $\mathcal{N}$ ein Numeroid und $\mathcal{M}$ ein $\mathcal{N}$-Numeroid mit strukturellen Homomorfismus $\varphi : \mathcal{N} \to \mathcal{M}$. Wenn $x_1, x_2, \ldots, x_n$ ein System von Elementen Aus $\mathcal{M}$ ist, dann existiert ein einziger Homomorfismus von $\mathcal{N}$- Numeroiden:

(9) $u: A[X_1, X_2, \ldots, X_n] \to \mathcal{M}$ so das $u(X_i) = x_i$

Beweis. Für n=1 wurde die Afirmation beweisen in Satz 2 Man nimmt an, die Aussage sei wohn fir n-1 . Dann existiert ein komutatives Diagramm

(10)


u' ist ein Homomorfismus von $\mathcal{N}$-Numeroiden lant mathematicher Induktion mit der Eigenschaft das $u'(X_i) = x_i$, $i = \overline{1,n}$

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FUNCTII PE STRUCTURI NUMEROIDALE

Rezumat

In această lucrare sînt prezentate 4 teoreme şi unele observaţii relative ŞA omomorfismul şi izomorfismul structurilor N-Numeroidale.

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REGULARITY AND UNIFORM CONTINUITY PROPERTIES OF COSINE AND SINE CLASS OF OPERATORS

Babescu Gh.

Let B(X) the algebra of bounded linear operators on a complex Banach spaces X.

In this note an class of cosine operators on X is meant to be a strongly continuous function on R, $C : R \longrightarrow B(X)$, fulfilling

(1) $C(s+t)+C(s-t) = 2C(s)C(t)$, $C(o) = I$, $(\forall)\ s,t \in R$.

Infinitesimal generator A of C is the closed and densely defined operator $A:D(A)\subset X \longrightarrow X$ given by

(2) $Ax = (d^2/dt^2)C(t)x|_{t=o} = \lim_{h\to 0}(2(C(h)-I)/h^2)x$

where $D(A)=\{x\in X: C(t)x$ is twice continuosly diferentiable$\}$.

To every strongly continuous cosine families on Banach spaces of operators we associate the strongly continuous sine family, S(t) $t \in R$ and the sets E_k, k = 1,2, defined respective by

(3) $S(t)x = \int_o^t C(s)x\, ds$, $t \in R$, $x \in X$

(4) $E_k = \{x \in X: C(\cdot)x \in C^{(k)}(R;X)\}$.

The cosine and sine class of operators strongly continuous C(t), S(t), $t \in R$, are related to abstract linear second order differential equations in the same manner that strongly continuous semigroups of bounded linear operators are related to abstract linear first order differential equations in Banach spaces.

The aim of this note is to investigate the regularity and uniformly continuity properties of cosine and sine class of operators.

For this we remind that B.Nagy [2] to show that the Cauchy problem of second order

(4) $u''(t) = Au(t)$, $u(o) = x \in E_2$, $u'(o) = y \in E_1$

where $u:R\to X$, $u\in C^2(R,X)$, $A:D(A)\subset X\to X$, $u(t)\in D(A)$ has unique solution $u(t;x,y)$ related to traiectory of $C(t)$ and $S(t)$, $t\in R$ by relation $u(t;x,y) = C(t)x + S(t)y$.

At the same time C.C.Travis and G.F.Webb [3] to show that every solution of inhomogeneous problem

$$(5)\quad u'' = Au(t) + f(t),\ t\in R,\ u(o) = x\in E_2,\ u'(o) = y\in E_1$$

where $f\in C^0(R,X)$, $u(\cdot)\in C^2(R,X)$, $u(t)\in D(A)$, has representation

$$(6)\quad u(t) = C(t)x + S(t)y + \int_0^t S(t-s)f(s)ds.$$

The function u from (6) has the properties:

Proposition 1. Let by $K\subset R$ compact and $u,f\in C^0(R,X)$. Then

1) $u\in C^2(R;X)$ if and only if

$$(7)\quad \lim_{h\to o}(1/h^2)(u(t+h)-2u(t)+u(t-h)) = u''(t),\ \text{uniform on } K$$

2) exist

$$(8)\quad \lim_{h\to o}(1/h^2)\left(\int_t^{t+h}S(t+h-s)f(s)ds + \int_t^{t-h}S(t-h-s)f(s)ds\right) = f(t),$$

uniform on K.

3) if $f(s)\in D(A)$, $s\in R$ and $Af(s)\in C^0(R;X)$ then

$$(9)\quad \lim_{h\to o}(2/h^2)(C(h) - I)f(s) = Af(s) \text{ uniform on K.}$$

The proof of this proposition is based fundamental properties of strongly continuous cosine class of operators.

The following theorem establishes the conditions of f which insure that the function u to give by (6) satisfy (5).

Theorem 1. If A is the infinitestimal generator of a strongly continuous cosine family $C(t)$, $t\in R$, defined by (2) and $f\in C^0(R,X)$ then every function $u:R\to X$, give by (6) is a solution of the initial value problem (5) if and only if the function $g:R\to X$, defined by relation

$$(1o)\quad g(t) = \int_0^t S(t-s)f(s)ds,$$

has one from the following properties

i) $g\in C^2(R;X)$

ii) $g(t)\in D(A)$ and $Ag(t)\in C^0(R;X)$, $(\forall)t\in R$.

Proof. i) If the problem (5) has a solution u for $x\in D(A)\subset E_2$, and $y\in E_1$, then this is give by (6). Now because that $g(t) = u(t) - S(t)y - C(t)x$ and $u,C,S\in C^2(R;X)$, we have $g\in C^2(R;X)$.

Conversely, we assume that g give by (1o) has the property i) and show that $g(t)\in D(A)$, $t\in R$, $Ag(t) = g''(t) - f(t)$, $t\in R$ and for $x\in D(A)$, $y\in E_1$ the function

(11) $u(t) = C(t)x + S(t)y + g(t)$, $(\forall)\ t \in R$

is a strongly solution of the problem (5).

The fact that $g(t) \in D(A)$, $t \in R$, to result from the identities:

$$S(s+t) + S(s-t) = 2S(s)C(t),$$

$$(2/h^2)(C(h) - I)g(t) = (1/h^2)(g(t+h) - 2g(t) + g(t+h)) -$$

$$- (1/h^2)(\int_t^{t+h} S(t+h-s)f(s)ds + \int_t^{t-h} S(t+h-s)f(s)ds).$$

Since $g \in C^2(R;X)$, while at limit is obtained equality, to result that i) holds.

ii) In proof of ii) we make us of the fact that every solution u of problem (5) has property $u(t) \in D(A)$, $t \in R$, $Au(t) \in C^0(R;X)$ and if $x \in D(A)$, $y \in E_1$, then $C(t)x, S(t)y \in D(A)$, $t \in R$ and $AC(t)x, AS(t)y \in C^0(R;X)$, therefore $g(t) = u(t) - C(t)x - S(t)y \in D(A)$, $t \in R$ and $Ag(t) \in C^0(R;X)$. Conversely, let $g(t) \in D(A)$, $t \in R$ and $Ag(t) \in C^0(R;X)$ and we show that for $x \in D(A)$, $y \in E_1$, the vector function (6) is solution for the problem (5). Since $u(t) \in D(A)$, $t \in R$ and $Au(t) \in C^0(R;X)$ we have that $u(t)$ $C^2(R;X)$, t R and exist equality

$(1/h^2)(u(t+h)-2u(t)+u(t-h)) = (2/h^2)(C(h)-I)C(t)x+(2/h^2)(C(h)-I)S(t)y+(1/h^2)\int_t^{t+h} S(t+h-s)f(s)ds+(1/h^2)\int_t^{t-h} S(t-h-s)f(s)ds+(2/h^2)(C(h)-I)\int_0^t S(t-s)f(s)ds.$

Here make $h \to o$, in basis properties 2),3) of the proposition 1, we have

$A\,C(t)x+AS(t)y+f(t)+A\int_0^t S(t-s)f(s)ds = Au(t)+f(t)$, $t \in K$

that is $u(t) \in C^2(R;X)$ and $u''(t) = Au(t)+f(t)$, $t \in R$.

The following theorem gives necessary and sufficient conditions that a strongly continuous cosine family be continuous in the operator norm at $t = o$.

Theorem 2. If $C(t)$, $t \in R$ is a strongly continuous cosine class of operators with infinitesimal generator A, and $S(t)$, $t \in R$ is associated class of the sine operators, then the following are equivalence:

(I) $\lim_{t \to o} C(t) = I$, in $B(X)$

(II) $\lim_{t \to o}(S(t))/t = I$, in $B(X)$

(III) $A \in B(X)$

(IV) $C(t)(X) \subset E_1$, $t \in I \subset R$

(V) $S(t)(X) \subset$ (A), $AS(t) \in C^0(I;X)$, $I \subset R$.

Proof. (I)⇒(II) hold since for $x\in X$ and $|t|>0$, we have

$$\|(S(t)x)/t-x\| = t^{-1}\|\int_0^t (C(s)x-x)ds\| \leq \sup_{0\leq s\leq t}\|C(s) - I\| \cdot \|x\| .$$

(II)⇒(III) hold since for t sufficiently small, $\|(S(t))/t-I\|<1$ and thus $S(t)^{-1}\in B(X)$. For $t=s$, from

$C(s+t)-C(s-t) = 2AS(s)S(t)$, $s,t\in R$ next that

$C(2t)-I = 2AS(s)S(t)$, that is $A = 2^{-1}(C(2t)-I)S(t)^{-1}S(t)^{-1}\in B(X)$

(III)⇒(IV) hold since $D(A)\subset E_1$.

(IV)⇒(V). Let $t_0,t_1\in(a,b) = I\subset R$ and $x\in X$. Then from

$S(s-t) = S(s)C(t) - S(t)C(s)$, $s,t\in R$ next that

$S(t_1-t_0)x = S(t_1)C(t_0) - S(t_0)C(t_1)x$, $x\in X$.

Since $C(t_0)x, C(t_1)x\in E_1$ and $S(t)(E_1)\subset D(A)$, $t\in R$, folows that $S(t_1-t_0)(X)\subset D(A)$ and so $S(t)(X)\subset D(A)$, for $t\in(a-b,b-a)$. Since A is closed and $S(t)$ is bounded, $AS(t)$ is closed and by the closed graph theorem, $AS(t)\in B(X)$, for $t\in(a-b,b-a)$. Habituate to show that $AS(t)x$ is continuous for $t\in(a-b,b-a)$ and so implies hold.

(V)⇒(I). From the properties cosine and sine class of operators and from asumptions deduced that $S(t)(X)\subset D(A)$, and $AS(t)$ is strongly continuous on $(a-b,b-a)$. Again the clased graph theorem yields $AS(t)\in B(X)$ for $t\in(a-b,b-a)$. As $AS(t)$ is strongly continuous on $(a-b,b-a)$ the Principle Uniform Boundedness implies the existence of a constant $M\geq 0$ such that $\|AS(t)\|\leq M$, $t\in I_1\subset((a-b)/2,(b-a)/2)$, that is ([1]):

$$\|C(t)x-x\| = \|A\int_0^t S(s)xds\| = \|\int_0^t AS(s)xds\| \leq |t|M\|x\| ,$$

and thus the proof is complete.

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PROPRIETATI DE REGULARITATE SI UNIFORM CONTINUITATE PENTRU CLASELE DE OPERATORI COSINUS SI SINUS

Rezumat.

In lucrare se dau proprietăţi de regularitate şi uniform continuitate pentru clasele de operatori cosinus şi sinus în algebra B(X).

Seminarul de Matematică şi Fizică
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SUR QUELQUES EQUATIONS DANS $\tilde{\mathcal{C}}_T^s$-SEMIGROUPS

Goleţ I.

Soit $\mathcal{R} = \{F : R \longrightarrow [0,1]$, nondecroissante et continue à gauche$\}$. Dans le cas des $\mathcal{C}_T$ - semigroups on a demontré dans [4] que pour toute t - norme stricte T le semigroup $(\mathcal{R}, \mathcal{C}_T)$ contient des éléments qui ne peuvent pas se décomposer. L'absence de la loi de décomposition nous indique l'étude d'équations de la forme suivante :

Étant donnés F et $H \in \mathcal{R}$ on demande de trouver $G \in \mathcal{R}$ tel que

(1) $\quad \tilde{\mathcal{C}}_T^s(F,G) = H$.

Soit $\mathcal{D} = \{F \in \mathcal{R} \mid F(0) = 0$ et $F(\infty) = 1\}$. D'abord on va montrer que pour touts $F,H \in \mathcal{D}$, T une t-norme continue à gauche [6] et S une opération continue sur R_+ [5] l'inéquation

(2) $\quad \tilde{\mathcal{C}}_T^s(F,G) \leq H$

admets une solution maximale $G \in \mathcal{D}$.

Théorème 1. Soit T une t-norme continue à gauche et s une opération continue sur R_+ .

Pour tous $x,y,z \in R_+$ on définie :

(3) $M(y,z) = \sup\{a \in [0,1] \mid T(F(z),a) \leq H(s(y,z))\}$;

(4) $G^*(y) = \inf\limits_{z \in R_+} \{M(y,z)\}$;

(5) $G(x) = \begin{cases} 0 & \text{pour } x \leq 0 \\ \lim\limits_{y \nearrow x} G^*(y) & \text{pour } x > 0. \end{cases}$

Alors, nous avons :

a) $G \in \mathcal{D}$ est une solution maximale dans $\mathcal{D}$ de l'équation (2) c.à.d. pour tout $E \in \mathcal{D}$, si $\tilde{\mathcal{C}}_T^s(F,E) \leq H$ alors $E \leq G$;

b) Il existe une fonction $E \in \mathcal{D}$ telle que $\tilde{\mathcal{C}}_T^s(F,E) = H$ si et seulement si $\tilde{\mathcal{C}}_T^s(F,G) = H$.

On va donner dans la suite, des conditions nécessaires pour l'existence de la fonction G satisfaisant à l'équation (1).

Théorème 2. Soit T une t-norme continue à gauche, s une opération continue et stricte croissante sur $[0,\infty)$ telle que $s(x,y) \geqslant x + y$ et soit $F,H \in \mathcal{D}$.

On suppose que $t_1 = \inf\{x \mid F(x) = 1\}$ et $t_2 = \inf\{x \mid H(x) = 1\}$ existent et sont finis.

Alors une condition necessaire pour l'existence d'une fonction $G \in \mathcal{D}$ satisfaisant à l'équation

$$\tilde{\mathcal{C}}_T^s(F,G) = H \tag{6}$$

est

$$H(s(t_1 - \delta, t_2 - \delta)) \geqslant F(t_1 - \delta) \tag{7}$$

pour tout $0 < \delta < t_1$.

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ASUPRA UNOR ECUATII IN $\tilde{\mathcal{C}}_T^s$-SEMIGRUPURI

Rezumat

In această lucrare este construită soluţia G a unei ecuaţii de forma $(F,G) = H$ în cazul cînd ea există şi este dată o condiţie necesară pentru existenţa lui G.

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ABOUT THE HOMOMORPHISMS OF THE ADDITIVE GROUP $\mathbb{R}$ IN SOME GROUPS OF REALE LINEAR BINARY RELATIONS

Burlacu E., Crstici B., Neagu M.

O.St.Berţi and E. Burlacu [1] considered the following law of composition in $\mathbb{R}^3$, in connection with real linear binary relations:

(1) $(a,b,c)\,(a',b',c') = (aa', -bb', a'c-bc')$

The set $\mathbb{R}^3$ endowed with the usual addition and scalar multiplication and with the product of two elements (vectors) given by (1) is a linear algebra over $\mathbb{R}$ of the third order which we denote by $\mathfrak{A}_3$.

In what follows we shall determinate the homomorphismus of the additive group $\mathbb{R}$ in the following thrse subgroups of the maximal group of $\mathfrak{A}_3$.

$$G_1 = \{(a,b,c)\in\mathbb{R}^3 \mid a\neq 0,\ b\neq 0,\ c=k(a+b),\ k\neq 0 \text{ a given constant}\}$$

$$G_2 = \{(a,b,c)\in\mathbb{R}^3 \mid a\neq 0,\ b\neq 0,\ a+b=0\}$$

$$G_3 = \{(a,b,c)\in\mathbb{R}^3 \mid a\neq 0,\ b\neq 0,\ c=0\}$$

Thus we have the functional equation

(2) $f(x+y)=f(x)f(y)$, $(\forall)\ x,y\in\mathbb{R}$

for the function $f:\mathbb{R}\longrightarrow G_i$, $i=1,2,3$

1. In the case of the group G_1 the unknown function f must be of the form

$f(x) = (f_1(x), f_2(x), f_3(x))$, $f_i:\mathbb{R}\longrightarrow\mathbb{R}$, $i=1,2,3$, where

$k(f_1(x) + f_2(x)) = f_3(x)$, $(\forall)x\in\mathbb{R}$

For the scalar functions f_i we obtain the system of functional equations

(3) $f_1(x+y)=f_1(x)f_1(y)$, $f_2(x+y)=-f_2(x)f_2(y)$,

$f_3(x+y)=f_3(x)f_1(y) - f_2(x)\,f_3(y)$

We deduce the

Proposition 1. The homomorphism of the group $(\mathbb{R},+)$ in the group $(G_1,\cdot)$, where the dot denotes the product (1), are given by the function

(4) $f(x)= (\varphi(x), \psi(x), k(\varphi(x)-\psi(x))$

where φ and ψ are arbitrary homomorphisms of the additive group $\mathbb{R}$ in the multiplicative group $\mathbb{R}\setminus\{0\}$.

Corollary 1. The continuous homomorphisms of the group $(\mathbb{R},+)$ in the group $(G_1,\cdot)$ are given by the function

(5) $f(x)= (e^{ax}, -e^{bx}, k(e^{ax}-e^{bx}))$

Remark. The function (5) realises an isomorphism between the the group $(\mathbb{R},+)$ and his image through f (for each given $a \neq 0$ and $b \neq 0$).

2. In the case of the group G_2, the second equation from (3) coincide withe the first one, whereas the third equation becomes

(6) $f_3(x+y)= f_3(x)f_1(y) + f_1(x)f_3(y)$

It is known that the non-trivial solution of the functional equation

(7) $f_1(x+y) = f_1(x)f_1(y)$

(the first equation from the system (3)) is $\neq 0$, $(\forall)\ x\in\mathbb{R}$. Hence we can divide side by side the equations (6) and(7) and we obtain

$$\frac{f_3(x+y)}{f_1(x+y)} = \frac{f_3(x)}{f_1(x)} + \frac{f_3(y)}{f_1(y)}$$

Therefore we have the

Proposition 2. The homomorphisms of the group $(\mathbb{R},+)$ in the group $(G_2,\cdot)$ are given by the function

(4') $f(x)= (\varphi(x), -\varphi(x), \alpha(x)\varphi(x))$

where φ is an arbitrary homomorphism of $(\mathbb{R},+)$ in $(\mathbb{R}\setminus\{0\},\cdot)$ and α is an arbitrary endomorphism of the group $(\mathbb{R},+)$.

Corollary 2. The continuous homomorphisms of the group $(\mathbb{R},+)$ in the group $(G_2,\cdot)$ are given by the function

$f(x) = (e^{ax}, -e^{ax}, bx\, e^{ax})$

where a and b are arbitrary constants.

The remark after Corollary 1 is valid here too.

3. In the case of the group G_3 we have the system of FE

(8) $f_1(x+y) = f_1(x)f_1(y)$, $f_2(x+y)=-f_2(x)f_2(y)$

Therefore we have the

Proposition 3. The homomorphisms of the group $(\mathbb{R},+)$ in the

group $(G_3,\cdot)$ are given by the function

(4") $f(x) = (\varphi(x), -\psi(x), 0)$

where φ and ψ are arbitrary homomorphisms of the group $(\mathbb{R},+)$ in the group $(\mathbb{R}\setminus\{0\},\cdot)$

Corollary 3. The continuous homomorphisms of the group $(\mathbb{R},+)$ in the group $(G_3,\cdot)$ are given by the function

$$f(x) = (e^{ax}, e^{-bx}, 0)$$

where a and b are arbitrary constants.

The remark after the corollary 1 is valid here too.

4. In this part we shall study the functional equation (2) for the functions $f:\mathbb{R}\to\mathfrak{A}_3$. In a usual way we obtain (puting $x=y=0$)

(9) $f(0) = f^2(0)$

The equation (9) has in $\mathfrak{A}_3$ the following solutions

a) $f(0)= (1,-1,0)$

b) $f(0) = (0,0,0)$

c) $f(0) = (0,-1,\lambda)$, $\lambda\in\mathbb{R}$ arbitrary

d) $f(0) = (1,0,\mu)$, $\mu\in\mathbb{R}$ arbitrary

In the case a) the general solution of (2) is given by (4), (4') and (4") . In the case b) the general solution is the trivial solution $f(x) =(0,0,0)$, $(\forall)x\in\mathbb{R}$.

We obtain this result by puting in (2) $y=0$. Now we consider the case c)

By puting $y=0$ in (2) we obtain

$$f(x) = (0, f_2(x), - f_2(x)).$$

By replaring in (2) ,it results

$$f_2(x+y) = - f_2(x)f_2(y), \quad (\forall)x,y\in\mathbb{R}.$$

Hence

$$f_2(y)=-\varphi(x), \quad (\forall)\ x\in\mathbb{R}$$

where φ is an arbitrary non-trivial solution of the Cauchy functional equation

(9) $\varphi(x+y) =\varphi(x)\varphi(y)$, $(\forall)$ $x,y\in\mathbb{R}$

for the functions $\varphi:\mathbb{R}\to\mathbb{R}$.

Therefore , the general solution of the functional equation

(10) $f(x+y) = f(x)f(y)$, $(\forall)$ $x,y\in\mathbb{R}$,

$f:\mathbb{R}\to\mathfrak{A}_3$, $f(0) = (0,-1,\lambda)$

is

$$f(x) = (0,-\varphi(x),\lambda\varphi(x))$$

where λ is as described above.

In an analogous manner we conclude that in the case d) the function f must be of the form

$$f(x) = (f_1(x), 0, \mu f_1(x))$$

The scalar function $f_1 : \mathbb{R} \to \mathbb{R}$ must be solution of the functional equation

$$f_1(x+y) = f_1(x)\, f_1(y), \quad (\forall)\ x, y \in \mathbb{R}$$

Therefore, the general solution of the functional equation (1o), where $f(0)=(1,0,\mu)$ (instead of $f(0)=(0,-1,\lambda)$), is

$$f(x) = (\varphi(x), 0, \mu\varphi(x))$$

where φ is an arbitrary non-trivial solution of the functional equation(9).

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ASUPRA OMOMORFISMELOR GRUPULUI ADITIV $\mathbb{R}$ IN UNELE GRUPURI DE RELATII BINARE LINIARE REALE

Rezumat

Se studiază ecuaţia funcţională

$$f(x+y) = f(x)f(y), \quad (\forall)\ x, y \in \mathbb{R}, \quad f: \mathbb{R} \longrightarrow \mathfrak{A}_3$$

unde $\mathfrak{A}_3$ este o algebră liniară de ordinul 3 , care se obţine dacă în $\mathbb{R}^3$ definim produsul a două elemente astfel:

$$(a.b.c)(a',b',c') = (aa', -bb',\ a'c - bc')$$

Se obţin soluţiile :

$$f(x) = (\varphi(x), -\psi(x),\ k(\varphi(x)-\psi(x))$$

$$f(x) = (\varphi(x), -\varphi(x), \alpha(x)\varphi(x))$$

$$f(x) = (\varphi(x),\ -\psi(x),\ 0\ )$$

în cazul $f(0)=(1,-1,0)$, unde φ şi ψ sînt omomorfisme arbitrare ale grupului $(\mathbb{R},+)$ în grupul $(\mathbb{R}\setminus\{0\},\cdot)$, iar α este un endomorfism arbitrar al grupului $(\mathbb{R},+)$.

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ON THE REGULARIZATION OF SINGULAR PERTURBATIONS OF SOME LINEAR DIFFERENTIAL EQUATIONS IN CRITICAL CASES

Năslău P.

In this paper we shall construct the asymptotic expansion for the Cauchy problem

$$L_\varepsilon u \equiv \varepsilon \frac{du}{dx} - A(x)u(x,\varepsilon) = h(x),\ u(o,\varepsilon) = u^o \qquad (1)$$

for $\varepsilon \longrightarrow +0$ in some "critical" cases, extended the regularization method of S.A,Lomov [1] to the case when the eigenvalues of A(x) are multiple.

We impose fulfilled the following conditions:

Condition 1. $A(x)\ ,h(x) \in C^\infty[o,a]$, $o < a < \infty$

Where A(x) and h(x) are n x n respectivelly n x 1 dimensions matrices

Condition 2. The invariant subspace corresponding to each distinct eigenvalue $\lambda_i(x), i=\overline{o,s}$ of A(x) is n_i- dimensional , where n_i is algebraic multiplicity of tho respective eigenvalue and:

$$\lambda_o(x)=0, \lambda_i(x) \neq \lambda_j(x) \text{ for } x\in[o,a],\ i\neq j;\ i,j=\overline{o,s} \qquad (2)$$

We extend the function $u(x,\varepsilon)$ to the function $\tilde{u}(x,t,\varepsilon)$ as follows:

$$t=(t_1,t_2,\dots,t_s), \varphi(x,\varepsilon)=(\varphi_1(x,\varepsilon),\varphi_2(x,\varepsilon),\dots,\varphi_s(x,\varepsilon))$$

$$\varphi_i(x,\varepsilon)=\begin{cases} \frac{1}{\varepsilon}\int_0^x \lambda_i(\varkappa)\,d\varkappa\ , & x>o \\ o \quad \text{for } x=o \end{cases}, \quad \tilde{u}(x,t,\varepsilon)\Big|_{t:=\varphi(x,\varepsilon)} \equiv u(x,\varepsilon),$$

the problem (1) will be extended to the problem:

$$\tilde{L}_\varepsilon \tilde{u} \equiv \varepsilon \frac{\partial \tilde{u}}{\partial x} + D_\lambda \tilde{u} - A(x)\tilde{u} = h(x);\ D_\lambda = \sum_{j=1}^{s} \lambda_j \frac{\partial}{\partial t_j};\ \tilde{u}(o,o,\varepsilon) = u^o \qquad (3)$$

and the space C_x^n of the n-dimensional vectors depending of patameter x will be extended to the space:

$$U = \sum_{j=o}^{s} C_x^n \cdot e^{tj} \qquad , to = 0$$

If we take in C_x^n th bases: $\left\{ b_{k_i}^i \ , k_i=\overline{1,n_i},\ i=\overline{o,s} \right\}$ (4)

where $b_{k_i}^i$, $k_i=\overline{1,n_i}$ are the eigenvectors corresponding to λ_i , $i=\overline{o,s}$, then an arbitrary element $u\in U$ has the following expression:

$$u(x,t) = \sum_{i,j=o}^{s} \sum_{k_i=1}^{n_i} u_{k_i}^{ij} b_{k_i}^i e^{t_j} \ , \ t_o=o, \ u_{k_i}^{ij} \in C^\infty[o,a] \quad (5)$$

If we demande the solution of problem (3) in the form:

$$\tilde{u}(x,t,\varepsilon) = \sum_{r=o}^{\infty} \varepsilon^r u_r(x,t) \ , \ u_r(x,t) \in U \quad (6)$$

by identifying formal the coefficients near the same powers of ε we obtain:

$$L_o u_o = h(x), \ u_o(o,o)=u^o, \text{ where } L_o \equiv D_\lambda - A(x) \quad (7)$$

$$L_o u_i = -\frac{\partial u_{i-1}}{\partial x}, \ u_i(o,o)=0, \ i=1,2,\ldots \quad (8)$$

The spaces U and $C_x^n \cdot e^{t_j}$, $j=\overline{o,s}$ are invariant subspaces of the operator L_o and a basis of $\operatorname{Ker} L_o$ is the following one:

$$\left\{ b_{k_i}^i \ e^{t_i} \ , \ k_i=\overline{1,n_i} \ , \ i=\overline{o,s}, \ t_o=o \right\} \quad (9)$$

By considering the space U as a Hilbert space with the scalar product induced by (4) and his dual, biortogonal basis:

$$\left\{ b_{k_i}^{i*} \ , k_i=\overline{1,n_i} \ , \ i=\overline{o,s} \right\} \quad (10)$$

it results that the adjoint operator L_o^* to the operator L_o is $L_o^* = D_\lambda - A^*(x)$, $A^*(x)$ - adjoint matrix to $A(x)$, and that a basis for $\operatorname{Ker} L_o^*$ is : $\left\{ b_{k_i}^{i*} \ , \ k_i=\overline{1,n_i}, i=\overline{o,s} \ , t_o=o \right\}$ (11)

Let be : $L_o u(x,t) = h(x,t); h,u\in U, \ u(o,o)=u^o$ (12)

Theorem 1. If $A(x)$ fulfils the conditions 1 and 2 then the problem (8) is solvable in U if and only if:

$$h \perp \operatorname{Ker} L_o^* \quad (13)$$

Proof. $(h, b_{k_i}^{i*} \cdot e^{t_i}) = (L u, b_{k_i}^{i*} e^{t_i}) = (u, L_o^* b_{k_i}^{i*} e^{t_i}) = 0;$

Reciprocally , suppose that we have (13). Then by considering the vectors $b_{k_j}^j = (b_{1\,k_j}^{o\,j*}, \ldots, b_{n_o k_j}^{o\,j*}, b_{1\,k_j}^{1\,j*}, \ldots, b_{n_s k_j}^{s\,j*})$ of the basis (10) to the canonical basis and an arbitrary element $h(x,t)$ in the form:

$$h(x,t) = H(x)\hat{e}^t, \ H(x) = \left\{ h_{k_i}^{ij} \ , k_i=\overline{1,n_i}, i,j=\overline{o,n_i} \right\} \quad (14)$$

$$\hat{e}^t = (o, e^{t_1}, e^{t_2}, \ldots, e^{t_s})$$

the indices i and k_i denoting the rows and j the columns, the condition (13) on write:

$$\sum_{i=o}^{s} \sum_{k_i=1}^{n_i} h_{k_i}^{ij} b_{k_i k_j}^{ij\,*} = o \ , \ k_j=\overline{1,n_j} \ , \ j=\overline{o,s} \quad (15)$$

The arbitrary element $u\in U$ will be writen in the form:

$$u(x,t) = B(x) \ e^{M_n(t)} \ w(x) + C(x) e^t \ , \quad (16)$$

$M_n(t) = \mathrm{diag}\{t_o,\dots,t_o,t_1,\dots,t_1,\dots,t_s,\ \dots\ t_s\}$, $t_o=o$, the columus of B(x) are all the eigenvectors , w(x) being undetermined vector in C_x^n , whereas C(x) is a nx (s+1) matrix which follows to be determined.

By substituting h and u from (14) and (16) in (12) it results:

$$C(x)M_{s+1}(\lambda(x))\, e^{\hat{t}} + A(x)\,C(x)\cdot e^{\hat{t}} = H(x)e^{\hat{t}} \qquad (17)$$

where $M_{s+1}(t) = \mathrm{diag}\{o,t_1,t_2,\dots,t_s\}$. By substituting in (17) C(x) with B(x) D(x) and by multiplying by $B^{-1}(x)$ it results:

$$D(x)M_{s+1}(\lambda(x)) - M_n(\lambda(x))D(x) = B^{-1}(x)H(x) \qquad (18)$$

By considering the matrices D and $B^{-1}M$ in an analogous form with H from (14) with elements $\{d_{k_i}^{ij}\}$, $\{\alpha_{k_i}^{ij}\}$, respectively, it results from (18):

$$\alpha_{k_i}^{ii}(x) = o,\ k_i = \overline{1,n_i}\ ,\ i=\overline{o,s} \qquad (19)$$

$$d_{k_i}^{ij}(\lambda_j-\lambda_i) = \alpha_{k_i}^{ij}\ ,\ k_i=\overline{1,n_i}\ ,\ i\neq j,\ i,j=\overline{o,s} \qquad (20)$$

Therefore the algebraic system (18) can be solved under the condition (19) which is fulfilled in accordance to (15).

The elements of the unknown matrix are uniquely determined by (2o) for $i\neq j$ and arbitrary (e.q. zero) for i=j

The initial condition (12) because of (16) becomes:

$$B(o)\, e^{M_n(o)}\, w(o) + C(o) = u^o \qquad (21)$$

and since $\det(B(o)\, e^{M_n(o)}) \neq 0$ we can determine w(o).

Theorem 2. If the matrix A(x) fulfils the conditions 1 an 2 , $h\in\overline{U}$ fulfils the condition (13) then is unique the solution $u\in U$ which fulfils the condition

$$\frac{\partial u}{\partial x} \perp \ker L_o^* \quad \forall x \in [0,a] \qquad (22)$$

Proof. By differentiating u from (12) with respect to x, it results from (22):

$$w'(x) + R(x)\,w(x) + S(x) = 0 \qquad (23)$$

where:

$$S(x)=(\gamma_1^o,\gamma_2^o,\dots,\gamma_{n_o}^o,\gamma_1^1,\dots,\gamma_{n_s}^s),\ \gamma_{k_i}^i=((c^i)e^{t_i}, b_{k_i}^{i*}e^{t_i}),$$

c^i being the column-vectors of the matrix C(x), $R(x)=\mathrm{diag}\{R_{oo},\dots,R_{ss}\}$,

$$R_{ii}=\left\{r_{k_ik_j}^{ii} \mid r_{k_ik_j}^{ii} = ((b_{k_j}^i)e^{t_i}, b_{k_i}^{i*}e^{t_i});\ t=o, k_i,k_j=\overline{1,n_i}\right\}_{i=\overline{o,s}} \qquad (24)$$

From (23) and (21) it results that the solution (16) of (12) is completely determined.

Uniqueness:If we suppose that the problem (12)-(22)have two solutions u_1 and u_2, then the function $z= u_1-u_2$ verifies in U the conditions:

$z(x,t) = B(x)\, e^{M_n(t)}\, w(x)$; $w(o)=0$; $w'(x)+R(x)w(x)=0$, hence $u_1=u_2$

The preceding theorems allows to solve the problems (7) and (8). For the problem (7) one impose the condition (15).

In this case, the solution u_o will be determined in an analogous manner as in the theorem 1, whereas the vector $w_o(x)$ remains un determined.

But by the condition (22)for u_o which is necessary for solving (8) for i=1 one determines uniquelly $w_o(x)$too.In an analogous way one determines $u_\nu(x)$,hence a partial sum of the series(6):

$$u_{\varepsilon N}(x,t) = \sum_{f=0}^{N} \varepsilon^{\nu} u_{\nu}(x,t)$$

In an analogous way as in [1] one can give an asymptotic evaluation of the approximate solution $u_{\varepsilon N}(x,\varphi(x,\varepsilon))$of (1) andatheorem about the limiting process for the solution of(1) toward a certain solution of the "reduced"system.

Remark 1. The above results are valid for n =0 too.

Remark 2. Analogously as in[2] the results above can be extended to the case whwn in (1) appears $h(x,\varepsilon)$ in the place of $h(x)$.

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REGULARIZAREA PERTURBATIILOR SINGULARE ALE UNOR ECUATII DIFERENTIALE LINIARE IN CAZURI CRITICE

Rezumat

In această lucrare se extinde metoda regularizării a lui S.A. Lomov pentru ecuaţia (1) la cazul în care valorile proprii ale matricii A(x) sînt multiple.

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UBER EIN HYPERGRUPPOLD

Neuhaus N.

1. Es sei $(A,+,.)$ ein Hyperring[2] . Auf A betrachtet man die Hypeopertion $A\times A\to\mathcal{P}(A)$, $x*y = x+y+axy$ und erforscht Eigenschaften des Hypergruppoids [1] $(A,*)$.

2. Definition 2.1. Eine nichtleere Menge A ausgestattet mit einer Hyperoperation $x+y\subseteq A$, gennant Addition, und einer Operation $xy\in A$, gennant Multiplikation heisst Hyperring, wenn:

I. $(A,+)$ eine kanonische Hypergruppe [1] ist.

II. $(A,.)$ eine Halbgruppe ist, für die $xo = ox = o, \forall x\in A$.

III. die Multiplikation zweiseitig distributiv im bezug auf die Addition ist.

Definition 2.2. Ein Hyperring, der kein Ring ist, heisst eingentlich.

Definition 2.3. $s\in A$ heisst Skalarelement , wenn für $\forall x\in A$, $s+x$ aus einem einzigen Element besteht.

Satz 2.1. Die Menge S der Skalarelemente von $(A,+,.)$ ist ein zweiseitiges Hyperideal, [2].

3. Es ist $(A,+,.)$ ein Hyperring. $(A,*)$, wobei

$$x*y = x+y+axy,\ a\in A,\ \forall(x,y)\in A\times A \qquad (1)$$

ist offenbar ein Hypergruppoid.

Satz 3.1. Ist $(A,+,.)$ ein kommutativer Hyperring, so ist $(A,*)$ ein kommutatives Hypergruppoid mit Einselement:

$$e = 0\in(A,+).$$

Beweis. Aus

$x+y = y+x$ und $axy = ayx$ folgt $x*y = y*x$, und aus

$$0*x = o+x+aox = x = x*0 \text{ folgt } e = 0$$

Ist $(A,+,.)$ ein eigentlicher kommutativer Hyperring mit Einselement und bezeichnet man mit S bzw. S_* die Menge der Skalarelemente von $(A,+,.)$ bzw. $(A,*)$, so gelten folgende Sätze:

Satz 3.2. $S = S_*$

Beweis. Aus $(s,x)\in S \times A$ folgt sx, xs, $a(sx)\in S$. $s*x = x+(s+asx)$ ist folglich, für $\forall x\in A$, eine Menge mit einem einzigen Element, d.h. $s\in S$ und somit gilt $S\subseteq S_*$. Ist nun $s\in S_*$ so folgt es $\exists y\in A$ so, dass $s*x = x+s(1+ax) = y$, $\forall x\in A$, also $s\in S$ (denn $1+ax\notin S$, da $(A,+,\cdot)$ eigentlich ist) und somit gilt $S_*\subseteq S$.

Satz 3.3. $s_1*(u+s_2) = s_1*u+s_1*s_2 - s_1$ (2)

$$x*(u+s) = x*u+(1+ax)s = x*s+(1+ax)u, \quad (3)$$

$$(x+s_1)*(y+s_2) = x*y + s_1*s_2 + a(s_1y+s_2x) \quad (4)$$

$s, s_1, s_2\in S$, $\forall u, x, y\in A$.

Beweis. Man zieht in Betracht, dass $u+s_2$ aus einem einzigen Element besteht und wendet die Beziehung (1) an. Ähnlich, kann (3) hergeleitet werden. Für die Herleitung von (4) ist zu bemerken, dass $(s_1+x)(s_2+y) = s_1(s_2+y)+x(s_2+y)$ gilt.

Satz 3.4.

$$(s_1*x)*(s_2*y) = x*y+(s_1+s_2ax)*(s_2+s_2ay)+a[s_1(1+ax)y+ s_2(1+ax)x] \quad (5)$$

$$(s*x)*(s*y) = (1+sa)(x*y)+(s+sax)*(s+say)+sa^2xy \quad (6)$$

$\forall s, s_1, s_2\in S$, $\forall x,y\in A$.

Beweis. Man zeiht in Betracht, dass s_1+s_1ax, $s_2+s_2ay\in S$ und wendet die Beziehung (4) an, (6) folgt aus (5) für $s_1=s_2=s$

Satz 3.5. $x*(s*y) = (1+sa)(x*y)+s = (x*s)*y$ (7)

$\forall s\in S$, $\forall x,y\in A$.

Beweis. Man zieht in Betracht dass $s+say\in S$ und wendet (1) und (3) an.

Satz 3.6.

$$s*[x*(s*y) = (1+sa)(x*y)+(s+sax)*(s+say)+sa^2xy, \quad (8)$$

$\forall s\in S$, $\forall x,y,\in A$.

Beweis.

$s*[x*(s*y)] = s*[(1+sa)(x*y)+s] = s*\bigcup_{t\in x*y}\{s*(1+sa)t+s\}$. In der Folge wendet man (2) und (1) an.

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ASUPRA UNUI HIPERGRUPOID

Rezumat

Fiind dat un hiperinel $(A,+,\cdot)$ se studiază proprietăţi ale hipergrupoidului $(A,*)$ unde $x*y = x+y+axy$. $(x+y\subseteq A, xy\in A)$.

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THE TWO-SIDED LAPLACE-DIRICHLET TRANSFORMATION

Neamţu N.N.

By analogy with [3] we introduce the two-sided Laplace - Dirichlet transformation.

Let be $g : \mathbb{Z} \to \mathbb{R}$, $K \mapsto g(K)$ a function wich satisfies the following conditions:

(1) $g(K) \leq K$ for $K < 0$

(2) $g(K) \geq K$ for $K \geq 0$

Definition 1. We define original function f raported to the function g, o function $f : \mathbb{Z} \to \mathbb{C}$, $K \mapsto f(K)$ wich satisfies the following properties:

(3) $|f(K)| \leq M_1 e^{s_1 g(K)}$, $M_1 > 0$, $s_1 \geq 0$, $K \geq 0$

(4) $|f(K)| \leq M_2 e^{s_2 g(K)}$, $M_2 > 0$, $s_2 \geq 0$, $K < 0$

Definition 2. We name two-sided Laplace-Dirichlet transformation of the original function f raported to the function g mapping

(5) $F^*(\lambda) = (L(f \mid g))(\lambda) = \sum_{K=-\infty}^{\infty} f(K)\, e^{-\lambda g(K)}$, $\lambda \in \mathbb{C}$

When $f(K) = 0$ for $K < 0$, then the two-sided Laplace-Dirichlet transformation coincides with [3]

Theoreme 1. If $s_2 > s_1$, then two-sided Laplace-Dirichlet transformation is determined in the strip $s_1 < \mathrm{Re}\,\lambda < s_2$, is holomorphe in the strip $s_1' \leq \mathrm{Re}\,\lambda \leq s_2$, $s_1' > s_1$, $s_2' < s_2$, and its derivatives ave given by

(6) $(F^*(\lambda))' = \sum_{K=-\infty}^{\infty} (-g(K)\, f(K))\, e^{-\lambda g(K)}$

Demonstration. We have

(7) $F^{\#}(\lambda) = F^{\#}_{+}(\lambda) + F^{\#}_{-}(\lambda)$

where

(8) $F^{\#}_{+}(\lambda) = \sum_{K=0}^{\infty} f(K)\, \bar{e}^{\lambda g(K)}$, $F^{\#}_{-}(\lambda) = \sum_{K=-1}^{-\infty} f(K)\, \bar{e}^{\lambda g(K)}$

and to take into acount (1) - (4) and the convergence of geometric series the demonstration is finished.

In particular, putting $g(K) = K$, $K \in Z$

we obtain

(9) $F^{\#}(\lambda) = (L(f))(\lambda) = \sum_{K=-\infty}^{\infty} f(K)\, \bar{e}^{K\lambda}$.

The demonstration of the following basic properties of the two-sided Laplace transformation is obvious.

(10) $f(K) = \frac{1}{2\pi i} \int_{s-i\pi}^{s+i\pi} F^{\#}(\lambda)\, e^{K\lambda}\, d\lambda$, $K \in Z$, $s_1 < s < s_2$

(Laplace's inverse theoreme)

(11) $(L(f(K-l)))(\lambda) = \bar{e}^{l\lambda}\, F^{\#}(x)$, $l \in Z$ (delay theoreme)

(12) $F^{\#}(\lambda + \mu) = (L(e^{-K\mu} f(K)))(\lambda)$ (shift theoreme)

(13) $F^{\#}(\lambda)\, G^{\#}(\lambda) = (L(\sum_{m=-\infty}^{\infty} f(m)\, g(K-m)))(\lambda)$

(the two-sided Laplace transformation of the convolution of two functions $f(K) - g(K) = \sum f(m)\, g(K-m)$)

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TRANSFORMAREA LAPLACE-DIRICHLET BILATERALA

Rezumat

In lucrare se defineşte transformarea Laplace - Dirichlet bilaterală şi sînt arătate proprietăţile ei fundamentale.

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ON THE MULTIPLICATION OPERATOR ON FUNCTIONS SPACES

Găvruţă P., Ceauşu T.

In this paper we will investigate some properties of the multiplication operator on Banach function spaces. See also [1], [2], [5].

Let $(\Omega, \mathcal{T}, \mu)$ be a totally σ- finite non-atomic measure space. Let $\mathcal{M}(\Omega)$, $\mathcal{P}(\Omega)$ denote the class of measurable and non-negative measurable functions on Ω, respectively.

For $f \in \mathcal{M}(\Omega)$ we denote by f^* the nonincreasing rearrangement ao f and by f^{**} the average of f (see [3], [6])

A function norm $\varrho: \mathcal{P}(\Omega) \longrightarrow [0, \infty]$ is a mapping which satisfies the following condition for f,g in $\mathcal{P}(\Omega)$:

(i) $\varrho(f) = 0 \Leftrightarrow f = 0$ a.e. ; $\varrho(f+g) \leq \varrho(f) + \varrho(g)$;

$\varrho(af) = a\,\varrho(f)$ for any constant $a \geqslant 0$;

ii) $f \leq g$ a.e $\Rightarrow \varrho(f) \leq \varrho(g)$;

iii) $\forall E \in \mathcal{T}, \mu(E) < \infty$ and χ_E is the characteristic function of E , then $\varrho(\chi_E) < \infty$ and there exists a constant $C_E < \infty$ so that

$$\int f \chi_E \, d\mu \leq C_E \, \varrho(f)$$

for every f in $\mathcal{P}(\Omega)$

(iv) If $f_n \in \mathcal{P}(\Omega)$, $n \in \mathbb{N}$ so that $f_n \nearrow f$ a.e. , then $\varrho(f_n) \nearrow \varrho(f)$

Let L^ϱ denote the set of $f \in \mathcal{M}(\Omega)$ dor which $\varrho(|f|) < \infty$. Then , indentifying functions which differ at most on a set of measure zero, L^ϱ is a Banach space with the norm $\|f\|_\varrho = \varrho(|f|)$, for lach , $f \in L^\varrho$. L^ϱ is called a Banach function space.

If ϱ verifies.

(v) $f^* = g^* \Rightarrow \varrho(f) = \varrho(g)$

then ϱ is called a rearrangement invariant norm and L^ϱ is a rearrangement invariant space. (r.i. space) . See [4] .

Let be T_φ the operator $T_\varphi f = \varphi f$.

THEOREM 1. Are equivalent the following

a) T_φ is bounded on L^s;

b) $\varphi \in L^\infty(\Omega)$

c) There exists $c>0$ so that:

(1) $(T_\varphi f)^{**} < c f^{**}$, for every $f \in L^s$.

PROOF. We prove that a) $\Rightarrow$ b) We suppose that T_φ is bounded on L^s:

(2) $\|\varphi f\|_s \leq \|T_\varphi\| \cdot \|f\|_s$, $f \in L^s$

We prove that $\|\varphi\|_\infty \leq \|T_\varphi\|$. If $\|\varphi\|_\infty > \|T_\varphi\|$ we can find $\varepsilon>0$ and $M \in \mathcal{T}$ so that $0 < \mu(M) < \infty$ and $M \subset \{x \in \Omega : |\varphi(x)| \geq \|T_\varphi\| + \varepsilon\}$ Then we have $|\varphi| \chi_M \geq (\|T_\varphi\| + \varepsilon)\chi_M$ and from (ii) it follows

$$\|\varphi \chi_M\|_s \geq (\|T_\varphi\| + \varepsilon)\|\chi_M\|_s > \|T_\varphi\| \, \|\chi_M\|_s$$

what is in contradiction with (2).

We prove that b) $\Rightarrow$ c) . Suppose that $\varphi \in L^\infty(\Omega)$ and $f \in L^s$. We prove that

(3) $(T_\varphi f)^{**}(t) \leq \|\varphi\|_\infty f^{**}(t)$, $t>0$

If $0 < t \leq \mu(\Omega)$, by Lemma 3.17 (iii) [6] we can find a set E_t in $\mathcal{T}$ such that $\mu(E_t) = t$ and

$$\int_0^t (\varphi f)^*(s)\,ds = \int_{E_t} |\varphi(x) f(x)|\,dx$$

It follows

$$(\varphi f)^{**}(t) = \frac{1}{t}\int_{E_t} |\varphi(x) f(x)|\,dx \leq \|\varphi\|_\infty f^{**}(t)$$

It $\mu(\Omega) < \infty$ and $t > \mu(\Omega)$ the proof. of (3) is the same.

We prove that c) $\Rightarrow$ b) Suppose that (1) holds. We prove that $\|\varphi\|_\infty \leq c$. If $\|\varphi\|_\infty > c$ we can find $\varepsilon>0$ and $M \in \mathcal{T}$ so that $0 < \mu(M) < \infty$ and $M \subset \{x \in \Omega : |\varphi(x)| > c+\varepsilon\}$. We prove that

(4) $(\varphi \chi_M)^{**}(t) > c\,\chi_M^{**}(t)$, $t>0$

We have

$$\chi_M^{**}(t) = \begin{cases} 1 & \text{if } t \leq \mu(M) \\ \dfrac{\mu(M)}{t} & \text{if } t > \mu(M) \end{cases}$$

Suppose $t > \mu(M)$. Then $(\varphi\chi_M)^*(t) = 0$ It follows $(\varphi\chi_M)^{**}(t) =$

$$= \frac{1}{t}\int_0^{\mu(M)} (\varphi\chi_M)^*(s)\,ds \geq \frac{1}{t}\int_M |\varphi(x)|\,\chi_M(x)\,ds > (c+\varepsilon)\,\frac{\mu(M)}{t}$$

Suppose $t \leq \mu(M)$ .Then fos $s < t$ we have

$$(\varphi \chi_M)^* (s) > (c+s)$$

Hence $(\varphi \chi_M)^{**} (t) = \frac{1}{t} \int_0^t (\varphi \chi_M)^* (s)\, ds > c+\varepsilon$

We prove that b) $\Rightarrow$ a) Suppose $\varphi \in L^\infty(\Omega)$ and $f \in L^\varrho$. It follows $|\varphi(x)| \leq \|\varphi\|_\infty$ a.e.

Hence $|\varphi(x)|\,|f(x)| \leq \|\varphi\|_\infty |f(x)|$ a.e.

From (ii) it follows $\|T_\varphi f\|_\varrho \leq \|\varphi\|_\infty \|f\|_\varrho$

THEOREM 2. Let be L^ϱ r.i. space . Let be $\varphi \in L^\infty(\Omega)$, $D = \{x \in \Omega : |\varphi(x)| \leq 1\}$ and $f \in L^\varrho$. Then the following are equivalent

a) $\|T_\varphi^n f\|_\varrho \leq \|f\|_\varrho$

b) $\chi_D f = f$

c) $(T_\varphi^n f)^{**} \leq f^{**}$

PROOF. We prove that a) $\Rightarrow$ b) Let be $a > 1$ and $E_a = \{x \in \Omega : |\varphi(x)| \geq a\}$. We have

$$\varphi^n f \geq a^n f \chi_{E_a}$$

Hence $a^n \|f \chi_{E_a}\|_\varrho \leq \|\varphi^n f\|_\varrho \leq \|f\|_\varrho$. For $f \neq 0$ it follows $f\chi_{E_a} = 0$, $a > 1$. It follows $f \chi_D = f$.

We prove that b) $\Rightarrow$ c) . If $0 < t \leq \mu(\Omega)$ from Lemma 3.17(iii) [6] there exists $E_t \in \mathcal{T}$, $\mu(E_t) = t$ and

$$\int_0^t (\varphi^n f \chi_D)^*(s)\, ds = \int_{E_t} |\varphi^n(x) f(x)|\, \chi_D(x)\, dx$$

It follows

$$(\varphi^n f)^{**} (t) = (\varphi^n f \chi_D)^{**} (t) = \frac{1}{t} \int_0^t (\varphi^n f \chi_D)^* (s)\, ds$$

$$\leq \frac{1}{t} \int_{E_t \cap D} |f(x)|\, dx \leq f^{**}(t)$$

If $\mu(\Omega) < \infty$ and $t > \mu(\Omega)$ the proof is the same.

For c) $\Rightarrow$ a) see [4] .

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ASUPRA OPERATORULUI DE MULTIPLICARE PE SPAŢII DE FUNCŢII

Rezumat

In această lucrare se studiază unele proprietăţi ale operatorului de multiplicare pe spaţii Banach de funcţii, extinzîndu-se unele rezultate din [2] şi [1] .

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L'ESPACE LINÉAIRE TANGENT GÉNÉRALISE D'ORDRE DEUX

Radu F.

Soinet $D \xrightarrow{p} M$ et $Z^1 \xrightarrow{q} M$ deux faisceaux d'algèbres sur l'espace topologique M tels que, quel soit $x \in M$, Z^1_x est une sous - algèbre de l'algèbre D_x ($D_x = p^{-1}_{(x)}$, $Z^1_x = \bar{q}^1(x)$)

Définition 1 Z^1_x est une algèbre stationnaire d'ordre un de l'algèbre D_x s'il existe un système d'éléments $\{u^i_x, i=1,2,\ldots,n\} \subset D_x$. ayant les propriétés suivantes : 1° quel que soit $f_x \in D_x$ il existe $a_i \in \mathbb{R}$, $i \in \{1,2,\ldots, n\}$ tel que $f_x - a_i u^i_x \in Z^1_x$. 2° quels que soient $a_i \in \mathbb{R}$, $i =1,2,\ldots,n$ ($\sum a_i^2 \neq 0$) alors $a_i u^i_x \notin Z^1_x$.

Proposition 1 Étant donnée l'élément $f_x \in D_x$ les nombres réels $a_i \in \mathbb{R}$, $i=1,\ldots,n$ qui possèdent la propriété $f_x - a_i u^i_x \in Z^1_x$ sont d'une focon unique détérminés,.

Démonstration D'après la définition 1 il existe $a_i \in \mathbb{R}$, $i = 1,\ldots,n$ de sorte que $f_x - a_i u^i_x \in Z^1_x$ On considère $b_i \in \mathbb{R}$, $i = 1,\ldots,n$ tels que $f_x - b_i u^i_x \in Z^1_x$, alors $(f_x - a_i u^i_x) - (f_x - b_i u^i_x) = (a_i - b_i) u^i_x \in Z^1_x$ et en utilisant la condition. 2° de la définition 1 il en résulte que $a_i = b_i$ $i \in \{1,\ldots,n\}$.

Définition 2 Sous - algèbre Z^2_x de l'algèbre D_x est appelé sous - algèbre stationnaire d'ordre deux si sont vérifiées les propriétés : 1° pour tout $f_x \in D_x$ il existe $a_{ij} \in \mathbb{R}$, $a_{ij} = a_{ji}$ $i,j \in \{1,\ldots,n\}$ tels que $f^1_x - \frac{1}{2} a_{ij} u^i_x u^j_x \in Z^2_x$. 2° quelsque soient $a_{ij} \in \mathbb{R}, \left(\sum_{i,j} a^2_{ij} \neq 0\right)$ $i,j \in \{1,\ldots,n\}$, $a_{ij} u^i_x u^j_x \notin Z^2_x$ 3°. $u^i_x \in Z^2_x$, $i \in \{1,\ldots,n\}$.

Proposition 2. Étant donnée l'élément $f_x \in D_x$, les nombres réels $a_{ij} \in \mathbb{R}$, i, j =1, ... n, qui possèdent la propriété : $f_x - \frac{1}{2} a_{ij} u_x^i \cdot u_x^j \in Z_x^2$ sont d'une facon unique détérminés.

Proposition 3. Soit $f_x \in D_x$, si $a_i \in \mathbb{R}$, $a_{ij} \in \mathbb{R}$, i,j $\in \{1,...n$ sont les nombres réels qui possèdent les propriétés , $f_x - a_i u_x^i = g_x \in Z_x^1$ et $f_x - \frac{1}{2} a_{ij} u_x^i u_x^j \in Z_x^2$ alors $g_x - \frac{1}{2} a_{ij} u_x^i \cdot u_x^j \in Z_x^2$

Démonstration Soient $b_{ij} \in \mathbb{R}$ tels que $g_x - \frac{1}{2} b_{ij} u_x^i \cdot u_x^j \in Z_x^2$

On obtient alors: $f_x - a_i u_x^i - \frac{1}{2} b_{ij} u_x^i u_x^j \in Z_x^2$ et puisque $f_x - \frac{1}{2} a_{ij} u_x^i \cdot u_x^j \in Z_x^2$, il résulte: $(f_x - \frac{1}{2} a_{ij} u_x^i u_x^j) - (f_x - a_i u_x^i - \frac{1}{2} b_{ij} u_x^i \cdot u_x^j) = a_i u_x^i + \frac{1}{2} (b_{ij} - a_{ij})$

$u_x^i \cdot u_x^j \in Z_x^2$. En tenant compte que $a_i u_x^i \in Z_x^2$ (la condition 3°. de la définition 2) on déduit: $(b_{ij} - a_{ij}) u_x^i \cdot u_x^j \in Z_x^2$ et donc $b_{ij} = a_{ij}$ d'après la condition 2.de la définition 2.

Corollaire Quel que soit $f_x \in D_x$ il existe les nombres réels a_i, $a_{ij} \in \mathbb{R}$ i,j $\in \{1,2,...,n\}$ unique détérminées tel que $f_x - a_i u_x^i - \frac{1}{2} a_{ij} u_x^i \cdot u_x^j \in Z_x^2$.

Remarque La proposition 1 et la proposition 2 détérminent les applications réels $d_i : D_x \to \mathbb{R}$, $d_{ij} : D_x \to \mathbb{R}$, i,j, $\in \{1,...,n\}$ définies de la facon suivante : soit $f_x \in D_x$ si $a_i \in \mathbb{R}$, i $\in \{1,...,n$ sont les nombres réels tels que $f_x - a_i u_x^i \in Z_x^1$ et a_{ij}, i,j $\in \{1,...,n\}$ sont les nombres réels telsque $f_x - \frac{1}{2} a_{ij} u_x^i \cdot u_x^j \in Z_x^2$ alors : $d_i (f_x) = a_i$, et $d_{ij}(f_x) = a_{ij}$.

Proposition 4 Les applications $d_i : D_x \to \mathbb{R}$, $d_{ij} : D_x \to \mathbb{R}$ i,j $\in \{1,...,n\}$ possèdent les propriétés suivantes :

1°. $d_i (af_x + bg_x) = a d_i(f_x) + b d_i (g_x)$, 1'. $d_{ij}(af_x + bg_x) = ad_{ij} (f_x) + bd_{ij} (g_x)$. 2°. $d_i (u_x^j) = \delta_i^j$

2'. $d_{ij}(u_x^k) = 0$, 3'. $d_{ij}(u_x^k \cdot u_x^s) = \delta_i^k \cdot \delta_j^k$, $\forall f_x, g_x \in D_x$, a,b $\in \mathbb{R}$, i,j, K,s $\in \{1,...,n\}$

Démonstration 1'. Soient $d_{ij}(f_x) = a_{ij}$, $d_{ij}(g_x) = b_{ij}$ alors $f_x - \frac{1}{2} a_{ij} u_x^i \cdot u_x^j \in Z_x^2$, $g_x - \frac{1}{2} b_{ij} u_x^i \cdot u_x^j \in Z_x^2$. On obtient $af_x + bg_x - \frac{1}{2}(a \cdot a_{ij} + b \cdot b_{ij}) u_x^i \cdot u_x^j \in Z_x^2$, donc $d_{ij}(af_x + bg_x) = aa_{ij} + bb_{ij} = ad_{ij}(f_x) + bd_{ij}(g_x)$.

Définition 3. On considère un homomorphisme d'algèbres $\varphi_x : D_x \to \mathbb{R}$. Les formes linéaires $d_i : D_x \to \mathbb{R}$ et $d_{ij} : D_x \to \mathbb{R}$, $i,j \in \{1,2,\ldots,n\}$ sont appélées les dérivées parcielles de la première ordre et respectivement les dérivées parcielles de la seconde ordre relatif à l'homomorphisme $\varphi_x : D_x \to \mathbb{R}$ si $d_i(f_x \cdot g_x) = \varphi_x(f_x) d_i(g_x) + \varphi_x(g_x) d_i(f_x)$ et $d_{ij}(f_x \cdot g_x) = \varphi_x(f_x) d_{ij}(g_x) + d_i(f_x) d_j(g_x) + d_j(f_x) d_i(g_x) + \varphi_x(g_x) d_{ij}(f_x)$, $\forall f_x, g_x \in D_x$.

Définition 4. l'espace linéaire $T_x^*(M) = D_x / Z_x^1$ (l'espace facteur de l'espace linéaire D_x relatif au sous - espace linéaire Z_x^1) est appelé l'espace linéaire des covecteurs tangents à l'espace topologique M dans le poit $x \in M$ et $T_x(M)$ le dual de $T_x^*(M)$ sera appelé l'espace linéaire tangent à l'espace topologique M en $x \in M$.

Définition 5 On considére $T_x'^*(M) = D_x / Z_x^2$ et $T_x'(M)$ le dual de $T_x'^*(M)$ L'espace linéaire $T_x(M) \oplus T_x'(M)$ est appelé l'espace linéaire tangent d'ordre deux à l'espace topologique M en $x \in M$. (On notera $T_x(M) \oplus T_x'(M)$ par $T_x^2(M)$).

Proposition 5 Soient les homomorphismes natrels d'espaces linéaires $d^1 : D_x \to T_x^*(M)$ et $d^2 : D_x \to T_x'(M)$ ($d^1 f_x$ ou $d^2 f_x$ - étant la classe d' equivalence de l'élément $f_x \in D_x$ en rapport avec la relation de congruence moduls Z_x^1 ou Z_x^2) Si $\{u_x^i, i=1,\ldots,n\}$ est le système d'éléments détérminés par la définition 1 alors $\{d^1 u_x^i, i = 1,\ldots,n\}$ et $\{d^2(u_x^i \cdot u_x^j), i,j \in \{1,\ldots,n\}, i \leq j\}$ sont les bases dans l'espace $T_x^*(M)$ et respectif $T_x'^*(M)$.

Démonstration Soit $\hat{u}_x \in T'^*_x(M)$, il existe $f_x \in D_x$ tel que $d^2 f_x = \hat{u}_x$ Il résulte de la définition 1 qu'il existe $a_{ij} \in \mathbb{R}$, $i,j \in \{1,\dots,n\}$ qui possèdent la propriété $f_x - \frac{1}{2} a_{ij} u^i_x u^j_x \in Z^2_x$; alors $d^2(f_x - \frac{1}{2} a_{ij} u^i_x \cdot u^j_x) = \hat{0}' \in T'^*_x(M)$ et donc $d^2 f_x = \frac{1}{2} a_{ij} d^2(u^i_x \cdot u^j_x)$, $i \le j$, $i,j \in \{1,\dots,n\}$ On obtient : $T'^*_x(M) = \mathbb{R}(\{d^2(u^i_x \cdot u^j_x); i \le j, i,j \in \{1,\dots,n\}\})$ Si l'on supposera que $a_{ij} d^2(u^i_x \cdot u^j_x) = \hat{0}' \in T'^*_x(M)$ alors $a_{ij} u^i_x \cdot u^j_x \in Z^2_x$ et on obtient $a_{ij} = 0$ (de la définition 1) et donc $d^2(u^i_x \cdot u^j_x)$ $i \le j$, $i,j, \in 1,\dots,n$ est un systéme de vecteure linéaire indépendant.

Remarque On notera $\left\{\frac{\partial}{\partial x^i}, i \in \{1,\dots,n\}\right\}$ la base dual de la base $\{du^i_x, i = 1,2,\dots,n\}$ de l'espace $T^*_x(M)$ et $\left\{\frac{\partial^2}{\partial x^j \partial x^k} j \le k, j,k \in \{1,2,\dots,n\}\right\}$ la base dual de la base $\{d^2(u^j_x \cdot u^k_x), j \le k, j,k \in \{1,\dots,n\}\}$ Si $X^2_x = (X_x, X'_x)$ est un vecteur tangent d'ordre deux en $x \in M$ alor $X^2_x(d^1 f_x, d^2 f_x) = X_x(d^1 f_x) + X'_x(d^2 f_x)$ on peut écrire $X^2_x = a^i \frac{\partial}{\partial x^i} \oplus a^{ik} \frac{\partial^2}{\partial x^j \partial x^k}$, $j \le k, i,j,k \in \{1,\dots,n\}$ On peut définire $\overline{X}^2_x(f_x) = X_x(d^1 f_x) + X'_x(d^2 f_x)$.

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SPAŢIUL LINIAR TANGENT GENERALIZAT DE ORDINUL DOI

Rezumat

Bazîndu-ne pe notaţiunile de algebră staţionară de ordinul întîi şi de ordinul al doilea, în prima parte a lucrării se introduc noţiunile de derivate parţiale într-un punct $x \in M$. In partea a doua a lucrării se introduce noţiunea de spaţiu liniar tangent de ordinul al doilea într-un punct $x \in M$.

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REMARKS ON THE CHARACTERISTIC SPECTRUM OF A DIAGONAL OPERATOR

Topuzu E.

1. Introduction. Let X be a complex Banach space. We shall denote by $\mathcal{B}(X)$, or $\tilde{X}$, the Banach algebra of all bounded linear operators on X.

If $A \in \mathcal{B}(X)$, as usual, $\rho(A)$ and $\sigma(A)$ denote the resolvent set and respectively, the spectrum of operator A.

It is well known that the Cauchy problem

$$(A;\ x_0) \quad \begin{cases} \dot{x} = Ax \\ x(0)=x_0 \end{cases}$$

has a unique solution, for every $A \in \mathcal{B}(X)$ and $x_0 \in X$.

Moreover, this solution is given by

$$x(t,t_0) = e^{tA} x_0$$

The real number

$$\nu_A(x_0) = \limsup_{t\to\infty} \frac{\ln \|e^{tA} x_0\|}{t}$$

is usually called the characteristic number of $x(t,x_0)$ and set

$$\chi(A) = \{\nu_A(x) ;\ x \in X ,\ x \neq 0\}$$

is named the characteristic spectrum of the operator A.

The characteristic numbers associated to a linear differential system in a finite dimensional space were used for the first time, by A.M.Liapunov in the stability theory. Liapunov has been shown that, if $\dim X < \infty$, then $\chi(A) = \mathrm{Re}\,\sigma(A) = \{\mathrm{Re}\,\lambda,\ \lambda \in \sigma(A)\}$.

Generalizations to the infinite dimensional case, can be found in [1], [2], where similar conexions between $\chi(A)$ and $\sigma(A)$ for some classes of bounded operators are presented.

In the following, we shall make some remarks about conexior

between $\chi(A)$ and $\sigma(A)$, if A is a diagonal operator on ℓ^p.

2. Diagonal operators. Let X be the Banach space ℓ^p, $1 \leqslant p \leqslant \infty$, that is

$$\ell^p = \ell^p(\mathbb{N}) = \{ x=(x_n)_{n\geqslant 0} \; ; \; x_n \in \mathbb{C} \; , \|x\|_p = (\sum_{n=0}^{\infty} |x_n|^p)^{1/p} < \infty \}$$

$$\ell^\infty = \ell^\infty(\mathbb{N}) = \{ x=(x_n)_{n\geqslant 0} \; ; \; x_n \in \mathbb{C} \; , \|x\|_\infty = \sup_{n\geqslant 0} |x_n| < \infty \}$$

If $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_n, \ldots) \in \ell^\infty$, then the bounded operator $D_\alpha : \ell^p \to \ell^p$, $1 \leqslant p \leqslant \infty$, defined by

$$D_\alpha(x) = \alpha x = (\alpha_n x_n)_{n\geqslant 0} \; , \; (\forall) x \in \ell^p$$

is usually called [3] diagonal operator generated by α.

The following Lemmas will be used in the sequel:

Lemma 1 ([1]). If $A \in \mathcal{B}(X)$ and $\sigma_p(A) = \{\lambda \in \sigma(A); \; Ker(A-\lambda I) \neq \{0\}\}$, then $Re\,\sigma_p(A) \subset \chi(A)$.

Lemma 2 (the ℓ^2-case are presented in [3]). If D_α is a diagonal operator on ℓ^p, then the following equalities hold:

$$\sigma_p(D_\alpha) = \{\alpha\}$$
$$\sigma(D_\alpha) = \overline{\{\alpha\}}$$

where $\overline{\{\alpha\}}$ denote the closure in $\mathbb{C}$ of the set

$$\{\alpha\} = \{\alpha_0, \alpha_1, \ldots, \alpha_n, \ldots\}$$

Theorem 1 . Let $D_\alpha : \ell^p \to \ell^p$ a diagonal operator and $\lambda \in \sigma(D_\alpha) = \overline{\{\alpha\}}$.

(i) If for every $\varepsilon > 0$, $Re\{\alpha\} \cap (Re\lambda - \varepsilon, Re\lambda] \neq \emptyset$ then $Re\lambda \in \chi(D_\alpha)$

(ii) If there exists $\varepsilon_0 > 0$, such that $Re\{\alpha\} \cap (Re\lambda - \varepsilon_0, Re\lambda] = \emptyset$ then $Re\lambda \notin \chi(D_\alpha)$.

Proof. (i) Let us consider that

$$Re\{\alpha\} \cap (Re\lambda - \varepsilon, Re\lambda] \neq \emptyset \; , (\forall)\varepsilon > 0 .$$

If we can find $\alpha_{i_o} \in \{\alpha\}$ such that $Re\alpha_{i_o} = Re\lambda$, then by virtue of Lemmas 1- 2 , it results that $Re\lambda \in \chi(A)$.

If the above case is not true, then we can find a subsequence $(\alpha_{i_k})_k$, extract from $(\alpha_k)_{k\geqslant 0}$, with the property $Re\alpha_{i_k} \to Re\lambda$. Let $x \in \ell^p$ such that $x_i = 0$, if $i \neq i_k$ and $x_{i_k} \neq 0$.

Then

$$\|e^{D_\alpha t}\|_p = \|e^{\alpha_i t} x_i\|_p = \|e^{\alpha_{i_k} t} x_{i_k}\|_p > e^{(Re\lambda - \varepsilon)t} |x_{i_{k_o}}|$$

from where $\nu_{D_\alpha}(x) \geq \mathrm{Re}\,\lambda$.

On the other hand

$$\|e^{D_\alpha t}x\|_p \leq e^{\mathrm{Re}\lambda t}\|x\|_p$$

such that (i) results.

(ii) Let us suppose now that $\mathrm{Re}\{\alpha\} \cap (\mathrm{Re}\lambda - \varepsilon_o, \mathrm{Re}\lambda] = \emptyset$ and we note

$$A' = \{\alpha_i \in \{\alpha\};\ \mathrm{Re}\alpha_i < \mathrm{Re}\lambda - \varepsilon_o\}\ ,\ I' = \{i \in \mathbb{N}\ ;\ \alpha_i \in A'\}$$

$$A'' = \{\alpha_i \in \{\alpha\};\ \mathrm{Re}\alpha_i > \mathrm{Re}\lambda\}\ ,\ I'' = \{i \in \mathbb{N}\ ;\ \alpha_i \in A''\}$$

For every $x \in \ell^p$, $x \neq o$, we have two posibilities

a) There exists $i_o \in I''$ such that $x_{i_o} \neq o$ and thus

$$\|e^{D_\alpha t}x\|_p \geq e^{t\mathrm{Re}\alpha_{i_o}}|x_i|$$

from where

$$\nu_{D_\alpha}(x) \geq \mathrm{Re}\,\alpha_{i_o} > \mathrm{Re}\,\lambda\ .$$

b) If $i \in \mathbb{N}$ and $x_i \neq o$, then $i \in I'$. In this case

$$\|e^{D_\alpha t}x\|_p = \|(e^{\alpha_i t}x_i)_{i\geq o}\|_p < e^{(\mathrm{Re}\lambda - \varepsilon_o)t}\|x\|_p$$

from where

$$\nu_A(x) \leq \mathrm{Re}\lambda - \varepsilon_o\ .$$

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OBSERVATII ASUPRA SPECTRULUI CARACTERISTIC AL UNUI OPERATOR DIAGONAL

Rezumat

In lucrare se studiază relaţia dintre spectrul unui operator diagonal care acţionează într-un spaţiu Banach de şiruri, ^{p}, şi spectrul său caracteristic.

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SUR LES PROPRIÉTÉS LATICEALES DES CONGRUENCES

Urosu C.

Pour une algébre universelle $U=\langle A,F\rangle$, la latice des congruences $\mathcal{C}(U)$ a deux typ des translations élémentaires : A-translations et V-translations.

Si $\theta\in\mathcal{C}(U)$, une V-translation élémentaire est defini par l'application:

$$\psi_\theta : \mathcal{C}(U)\to\mathcal{C}(U)\ ,\ \psi_\theta(\rho)=\rho\vee\theta,\ (\forall)\rho\in\mathcal{C}(U)$$

Analoque, l'application:

$$\varphi_\theta : \mathcal{C}(U)\to\mathcal{C}(U)\ ,\ \varphi_\theta(\rho)=\rho\wedge\theta,\ (\forall)\rho\in\mathcal{C}(U)$$ est une l-translation élémentaire

Pour chaque V-translation élémentaire ont lieu les propriétés:

1) $\rho\subseteq\psi_\theta(\rho)$, $(\forall)\rho\in\mathcal{C}(U)$
2) $\psi_\theta(\rho)=\rho\vee\theta=(\rho\vee\theta)\vee\theta=\psi_\theta(\psi_\theta(\rho))$, $(\forall)\rho\in\mathcal{C}(U)$
3) $\rho_1\subseteq\rho_2\to\rho_1\vee\theta\subseteq\rho_2\vee\theta\Leftrightarrow\psi_\theta(\rho_1)\subseteq\psi_\theta(\rho_2)$, $(\forall)\rho_1,\rho_2\in\mathcal{C}(U)$
4) $\psi_\theta(\rho_1\vee\rho_2)=\psi_\theta(\rho_1)\vee\psi_\theta(\rho_2)$ et $\psi'_\theta : \mathcal{C}(U)\to[\theta)$ est une epimorphisme semilaticeal("V")
5) $\psi_\theta(\omega_U)=\theta,\ \psi_\theta(\zeta_U)=\zeta_U,\ \psi_{\omega_U}(\rho)=\rho,\ \psi_{\zeta_U}(\rho)=\zeta_U,\ (\forall)\rho\in\mathcal{C}(U)$

La correspondance $\rho\longmapsto\rho/\theta$, $(\forall)\rho\in[\theta)$ ou $\rho/\theta=\{([x_1]\theta,[x_2]\theta)\ /\ [x_1]\rho=[x_2]\rho\ ,\ x_1,x_2\in A\}$. est une bijection entre $[\theta)$ et $\mathcal{C}(U)$.

Si la latice $\mathcal{C}(U)$ est distributife, chaque V-translation est une endomorphisme.

La translation ψ_θ, $\theta\in\mathcal{C}(U)$, a les suivantes propriétés:

1) $\psi_\theta(\rho)\subseteq\rho$, $(\forall)\rho\in\mathcal{C}(U)$
2) $\psi_\theta(\rho)=\psi_\theta(\psi_\theta(\rho))$, $(\forall)\rho\in\mathcal{C}(U)$
3) $\rho_1\subseteq\rho_2\to\psi_\theta(\rho_1)\subseteq\psi_\theta(\rho_2)$, $(\forall)\rho_1,\rho_2\in\mathcal{C}(U)$
4) $\psi_\theta(\rho_1\wedge\rho_2)=\psi_\theta(\rho_1)\wedge\psi_\theta(\rho_2)$.

Donc la translation $\varphi'_\theta : \mathcal{C}(U) \to (\theta]$ est une omomorphisme semi-laticeal("$\wedge$") et la congruence induit une congruence dans chaque algébre $U/\varphi\wedge\theta$.

La translation φ_θ este compatibile avec l'operation"V" si et seulment si la congruence θ est un élément standard dans $\mathcal{C}(U)$.

La latice $\mathcal{C}(U)$ est une latice algébrique et donc est supérieure continue [(1)] :
si $C \subseteq \mathcal{C}(U)$ est dirije supérieur , alors pour chaque $\theta \in \mathcal{C}(U)$ a lieu la relation:

$$\theta \wedge (\vee C) = \vee \{\theta \wedge \varphi \mid \varphi \in C\} = \vee \varphi_\theta(C)$$

Les congruences minimales proprés des algébre U sont atomes dans la latice $\mathcal{C}(U)$.

Théorème 1. Si l'algébre universelles U a la latice $\mathcal{C}(U)$ diatirbutife et C est une grand nombre des congruences minimales proprés ainsi qie la congruence minimale propré $\varphi \notin C$, alors $\varphi \wedge (\vee C) = \omega_U$

Démonstration. Si φ est une congruence minimale propre de l'algébres U , alieu la relation: $(\varphi] = \{\omega_U, \varphi\}$

Parce que $\mathcal{C}(U)$ est une latice algébrique, a lieu la relation

$$\vee C = \vee \{\vee B \mid B \subseteq C \text{ et } B \in \mathcal{P}_f(C)\}$$

et donc
$$\varphi \wedge (\vee C) = \varphi \wedge \vee \{\vee B \mid B \subseteq C \text{ et } B \in \mathcal{P}_f(C)\}.$$

La latice $\mathcal{C}(U)$ este superieur continue et distributife et donc a lieu la relation:

$$\varphi \wedge (\vee C) = \vee \{\varphi \wedge (\vee B) / B \subseteq C \text{ et } B \in \mathcal{P}_f(C)\} = \omega_B . /$$

Observation. Si la latice $\mathcal{C}(U)$ este modulaire et $\theta, \varphi \in \mathcal{C}(U)$ alors la translation φ_θ est une isomorphisme entre $[\theta \wedge \varphi, \varphi]$ et $[\theta, \theta \vee \varphi]$; l'isomorphisme inverse est φ_φ . Chaque deux intervales transposes $[\theta, \varphi]$ et $[\mu, \lambda]$ dans $\mathcal{C}(U)$ ($\theta = \varphi \wedge \mu$ et $\lambda = \varphi \vee \mu$ ou $\mu = \theta \wedge \lambda$ et $\varphi = \theta \vee \lambda$) sont isomorphes [(1)].

Théorème.2. Si la latice des congruences sur une algébres universelles U est modulaire et relative complémenté, alors chaque congruance $\theta \neq \omega_U$ contenit une congruence minimale propre.

Démonstration. Soit $\theta \in \mathcal{C}(U)$ et $\theta > \omega_U$. Existent les congruences $\varphi_1 , \varphi_2 \in (\theta]$ ainsi que $\varphi_1 < \varphi_2$ et grand nombre $\{\varphi \in \mathcal{C}(U) / \varphi_1 < \varphi < \varphi_2\}$ est vide [(2)].

La latice $\mathcal{C}(U)$ este relative complementé et soit μ un complemente relatife de la congruence φ_1 dans $(\theta]$: $\mu \wedge \varphi_1 = \omega_U$ et $\mu \vee \varphi_1 = \theta$

Si $p = \mu \wedge \varphi_2$, alors les intervales $(p]$ et $[\varphi_1, \varphi_2]$ sont transposes parce que : $(\mu \wedge \varphi_2) \wedge \varphi_1 = \mu \wedge \varphi_1 = \omega_U$ et donc $p \wedge \varphi_1 = \omega_U$

Conformément de la modularité dans la latice $\mathcal{L}(U)$, si $\varphi_1 < \varphi_2$ alors $p \vee \varphi_1 = (\mu \wedge \varphi_2) \vee \varphi_1 = \varphi_2 \wedge (\mu \vee \varphi_1) = \varphi_2 \wedge \theta = \varphi_2$ et donc $p \vee \varphi_1 = \varphi_2$ La congruence p est un atome dans $\mathcal{L}(U)$ et résulte que p est une congruence minimale propre de l'algébre U. A lieu la relation $p < \theta$. Par dualité a lieu la suivante théorème:

Théorème 3. Si la latice $\mathcal{L}(U)$ este modulaire et relative complémenté, alors chaques congruences $\theta \neq \iota_U$ est contenu dans une congruence propre maximale de l'algébre U.

Consequence. Si la latice $\mathcal{L}(U)$ este modulaire et relative complemente alors pour chaque $\theta \in \mathcal{L}(U)$ éxiste une congruence maximale propre $\varphi \in \mathcal{L}(U)$, ainsi que l'algebre $U/\theta/\varphi/\theta$ este une algébre simple.

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ASUPRA PROPRIETATILOR LATICEALE ALE CONGRUENTELOR

Rezumat

In lucrare sînt studiate proprietăţi ale translaţiilor elementare în laticea congruenţelor unei algebre universale şi se obţin rezultate relativ la congruenţele minimale (maximale) proprii.

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ON THE MEASURABILITY OF RANDOM GEODESICS.

Petrişor E.

Let (Ω,K,P) a complete probability measure space and M a connected n-dimensional smooth manifold. Denote by (T,π,M) the tangent bundle of M, by $(L_S^2 T,q,M)$ the vector bundle of twice-covariant symmetric tensors, by $(P_S^2 T,q,M)$ the open subbundle of positive definite forms and by $J^r(P_S^2 T)$ the manifold of r-jets of sections in $P_S^2 T$. Consider in $C^\infty(M, J^r(P_S^2 T))$ the open compact topology and define in Rie(M)-the space of all C^∞-sections in $P_S^2 T$-the C^r-ordinary topology $\mathcal{O}_r$, as the weakest topology in which the usual map j^r: Rie(M)$\longrightarrow C^\infty(M, J^r(P_S^2 T))$ is continous. Let $\mathcal{B}(\mathcal{O}_r)$ the σ-algebra of Borel sets in Rie (M).

If $(M,\tilde{g})$ is a random riemannian manifold, then $\tilde{g}$ defines a random linear connection $\tilde{\nabla}$ on M [4] . Denoting by $\tilde{\Gamma}^i_{jk}(x,\omega)$ the Christoffel symbols of the second kind of $\tilde{\nabla}$ in a local chart of M, the random differential equation

$$\frac{d^2x^i}{dt^2} + \tilde{\Gamma}^i_{jk}(x,\omega)\frac{dx^k}{dt}\frac{dx^j}{dt} = 0 \qquad (1)$$

is called the geodesic equation of the random riemannian manifold$(M, \tilde{g})$. A solution of the equation (1) is called a random geodesic.

In the theory of random differential equations of great importance is to etablish if a solution $x: (-\varepsilon,\varepsilon) \times \Omega \longrightarrow M$ is or not a random function, i.e for any $t\in(-\varepsilon,\varepsilon)$ and any borel set $B \subset M$ $x_t^{-1}(B) \in K$.

Theorem 1. If $(M,\tilde{g})$ is a random riemannian manifold such that $\tilde{g}$ is a measurable map, $\tilde{g}$: $(\Omega,K)\longrightarrow(\mathrm{Rie}(M),\mathcal{B}(\mathcal{O}_1))$ then the solution of equation (1) with initial conditions $p\in M$, $v\in T_pM$, is a random function on M.

The proof is based on the following result in the theory of differential equations [1] :

Proposition . Let $\Delta \subset R \times R^m$, $\varphi, \psi: \Delta \longrightarrow R^m$ two continuous functions so that φ satisfies the Lipschitz condition $\|\varphi(t,x)-\psi(t,x')\| \leq L\|x-x'\|$ for every $(t,x),(t,x')\in\Delta$. If

$x(t)=(x^1(t),\dots,x^m(t))$, $y(t)=(y^1(t),\dots,y^m(t))$ are solutions of the differential equations $\frac{dx}{dt}=\varphi(t,x)$, $\frac{dy}{dt}=\psi(t,y)$ and $\|\varphi(t,x)-\psi(t,x)\|\leq\varepsilon$, $0\leq t\leq a$, $(t,x)\in\Delta$, then $\|x(t)-y(t)\|\leq\|x(0)-y(0)\|\cdot e^{Lt}+\frac{\varepsilon}{L}(e^{Lt}-1)$.

Proof of the Theorem 1. Let $p\in M$, $v\in T_pM$ and $D_p^v=\{c:[0,1]\longrightarrow M \mid c \text{ of class } C^\infty,\ c(0)=p,\ \dot{c}(0)=v\}$ endowed with open compact topology τ_0. Define the map $\mathcal{G}:(\mathrm{Rie}(M),\sigma_1)\longrightarrow(D_p^v,\tau_0)$ which associates to every riemannian metric g the geodesic c_g starting from $p\in M$ with $\dot{c}_g(0)=v$, $v\in T_pM$.

Let (U,X) a coordinate chart of M, $p\in U$, and $(TU,\bar{X})$ the associated coordinate chart in TM. For $w\in T_qM$, $q\in U$, $\bar{X}(w)=(x^1,x^2,\dots,x^n,x^{n+1},\dots,x^{2n})$. Then the geodesic equations in the coordinate chart (U,X) of (M,g), $g\in\mathrm{Rie}(M)$, are:

$$\frac{dx^i}{dt}=x^{i+n}$$
$$\frac{dx^{i+n}}{dt}=-\Gamma_{jk}^{g,i}\,x^{j+n}\,x^{k+n} \qquad (2)$$

$i,j,k=\overline{1,n}$.

Let $\varphi_g:\bar{X}(TU)\subset R^{2n}\longrightarrow R^{2n}$, $\varphi_g^i(x^1,x^2,\dots,x^n,x^{n+1},\dots,x^{2n})=x^{i+n}$, $\varphi^{i+n}(x^1,x^2,\dots,x^n,x^{n+1},\dots,x^{2n})=-\Gamma_{jk}^{g,i}\,x^{j+n}x^{k+n}$, $i=\overline{1,n}$.

Then the equation (2) may be writen: $\frac{dx}{dt}=\varphi_g(x)$

Let $I=[0,a]$, $0<a\leq 1$, so that $\mathcal{G}(g)=C_g\in W(I,U)=\{c\in D_p^v \mid c(I)\subset U\}$. If $g'\in\mathrm{Rie}(M)$, then $x(t)=(C_g(t),\dot{c}_g(t))$, $y(t)=(C_{g'}(t),\dot{c}_{g'}(t))$, $t\in I$, are solutions of the differential equations $\frac{dx}{dt}=\varphi_g(x)$, $\frac{dy}{dt}=\varphi_{g'}(y)$.

Choose an open set $G\subset TU$ around $x(I)$, such that $\bar{G}$ is compact. Then φ_g satisfies the Lipschitz condition on $\bar{X}(\bar{G})$ Since the Christoffel symbols depend only on the coefficients of the riemannian metric and their first partial derivatives we can take $g'\in\mathrm{Rie}(M)$ so close to g on $K=\pi(\bar{G})$, in C^1-ordinary topology, such that $\|\varphi_g(x)-\varphi_{g'}(x)\|$ to be assmall as we wish on $\bar{X}(G)$.

Hence for any metric $g'\in\mathrm{Rie}(M)$ sufficiently close to g, the above Proposition assures that $C_{g'}(I)\subset U$ that is, given $\in D^v$ such that $C_{g'}\in W(I.U)$ there is a compact set $K\subset M$

and an open set $V \subset \mathcal{J}^1(P, T)$ so that for any $g' \in W(K,V) = \{h \in Rie(M) \mid j^1 h(K) \subset V\}$, $C_{g'} \in W(I,U)$, i.e. $\mathcal{G}$ is a continuous map.

It follows that $\mathcal{G}$ is a measurable map according to the σ-algebras of Barels sets in $(Rie(M)\sigma_1)$ and $(D_p^v, \mathcal{C}_o)$.

In the space $C_p(M)$, of the paths in M starting from p, the σ-algebra of Borel sets generated by the open compact topology, coincides with the σ-algebra $\mathcal{C}$ generated by cylindrical sets [3].

On the other hand $D_p^v \subset C_p(M)$ and the σ-algebra generated by $\mathcal{C}_o = D_p^v \cap \mathcal{C}$, $\mathcal{B}(D_p^v \cap \mathcal{C})$, coincides with $D_p^v \cap \mathcal{B}(\mathcal{C})$ [2].

Hence, $\mathcal{B}(\mathcal{C}_o) = D_p^v \cap \mathcal{C} = \mathcal{C}_o$, where by $\mathcal{C}_o$ we denoted the σ-algebra generated by cylindrical sets in D_p^v.

We can conclude that $\mathcal{G} \circ \tilde{g} : (\Omega, K) \longrightarrow (D_p^v, \mathcal{C}_o)$ is a measurable map and therefore $\tilde{c}: [0,1] \times \Omega \longrightarrow M$ the solution of equation (1) with initial conditions $\tilde{c}(o,\omega) = p$, $\tilde{c}(o,\omega) = v$ for every $\omega \in \Omega$, is a random function.

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ASUPRA MASURABILITATII GEODEZICELOR ALEATOARE

Rezumat

In lucrare se consideră două tipuri de varietăţi riemanniene aleatoare pe care geodezicele aleatoare constituie un proces stochastic pe varietate.

Seminarul de Matematică şi Fizică
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UTILISATION OF THE NEUTRIE CALCULUS FOR THE INCLUSION OF SOME CLASSES OF FUNCTIONS THAT ARE NOT LOCALLY INTEGRABLE IN THE DISTRIBUTIONS SPACE.

Halic G.

In this paper we apply the methodes of Neutrix Calculus for the inclusion of some classes of functions that are not locally integrable in the Distributions space.In §1 we expose the rules of Neutrix Calculus, that are utilised in this work.

I

§1. We introduce some notations for the neutralised values of integrales, after that we establish the rules of calculus with this neutralised values

Let $]a,b[$ be an open interval on the real axis and $c \in]a,b[$. We note by N_c-(respectively by N_{c+}) , the neutrix formed of functions that are defined on an open left (respectively right)neighbourhood of c, and having at the left (respectively at the right) of point c, the limit zero. The notations N_{a+} and N_{b-} have analogue meanings.

In this work , the assertion " function φ is integrable from a to b " is used in a generalised sense(φ is integrable in proper sens on $]a,b[$, or the integral of φ on $]a,b[$ is a convergent improper integral).

We say that the function φ is integrable from N_{a+} to c, if the function

$$\int_\xi^c \varphi(x)\,dx$$

is neutralised by the neutrix N_{a+}.

According to the notations adopted in Neutrix Calculus, for the neutralised value of a function, we replace the variable ξ by the symbol which denotes the neutrix, and we obtain:

$$\int_{N_{a+}}^c \varphi(x)\,dx$$

In a similar way, we define the symbol

$$\int_c^{N_{b-}} \varphi(x)\,dx$$

Theorem 1. If the function φ, defined on $]a,b[$ is integrable from N_{a+} to c and from c to b, then it is integrable from N_{a+} to b, and the following relation takes place:

$$(1)\quad \int_{N_{a+}}^{b} \varphi(x)\,dx = \int_{N_{a+}}^{c} \varphi(x)dx + \int_{c}^{b} \varphi(x)\,dx$$

In similar hypotheses, the following relation takes place:

$$(1')\quad \int_{a}^{N_{b-}} \varphi(x)\,dx = \int_{c}^{N_{b-}} \varphi(x)\,dx + \int_{a}^{c} \varphi(x)dx$$

If the function φ is integrable from N_{a+} to c and from c to N_{b-} then we say that φ is integrable from N_{a+} to N_{b-} and we note:

$$(2)\quad \int_{N_{a+}}^{N_{b-}} \varphi(x)dx = \int_{N_{a+}}^{c} \varphi(x)dx + \int_{c}^{N_{b-}} \varphi(x)dx$$

We can show that the value of the right member of the equality (2) does not depend on the chosing of the point $c \in]a,b[$.

Theorem 2. If the function φ is integrable from a to b, then it is integrable from N_{a+} to b, from a to N_{b-}, from N_{a+} to N_{b-} and the respective integrals are equal to one another.

Theorem 3. If the function φ defined on $]a,b[$ is integrable from N_{a+} to b, then it is integrable from N_{a+} to N_{b-} and the two integrals are equal to one another.

If the function φ defined on $]a,b[$ is integrable from a to N_{b-} then it is integrable from N_{a+} to N_{b-} and the two integrals are equal to one another.

Theorem 4. If $c \in]a,b[$, and the function φ is integrable from a to b, then the following holds:

$$(3)\quad \int_{a}^{b} \varphi(x)dx = \int_{a}^{N_{c-}} \varphi(x)\,dx + \int_{N_{c-}}^{b} \varphi(x)dx$$

Theorem 5. If $c \in]a,b[$ and the function φ is integrable from N_{a+} to b, then the following holds:

$$(4)\quad \int_{N_{a+}}^{b} \varphi(x)\,dx = \int_{N_{a+}}^{N_{c-}} \varphi(x)dx + \int_{N_{c+}}^{b} \varphi(x)dx$$

If φ is integrable from a to N_{b-}, then the following holds:

$$(4')\quad \int_{a}^{N_{b-}} \varphi(x)\,dx = \int_{a}^{N_{c-}} \varphi(x)dx + \int_{N_{c+}}^{N_{b-}} \varphi(x)dx$$

Theorem 6. If $c \in]a,b[$ and the function φ is integrable from

N_{a+} to N_{b-} , then following holds:

$$(5)\qquad \int_{N_{a+}}^{N_{b-}} \varphi(x)dx = \int_{N_{a+}}^{N_{c-}} \varphi(x)dx + \int_{N_{c+}}^{N_{b-}} \varphi(x)dx$$

Theorem 7. If the neutrix N_{a+} contoins together with the function ν ,the function $\lambda\nu$, where $\lambda$is a real number, and if the function φ defined on $]a,b[$ is integrable from N_{a+} to b, then, for each real number α , the function $\alpha\varphi$ is also integrable from N_{a+} to b and the following relation tokes place:

$$(6)\qquad \int_{N_{a+}}^{b} (\alpha\varphi)(x)\,dx = \alpha\int_{N_{a+}}^{b} \varphi(x)dx$$

In similar hypotheses, holds

$$(6')\qquad \int_{a}^{N_{b-}} (\alpha\varphi)(x)\,dx = \alpha\int_{a}^{N_{b-}} \varphi(x)dx$$

Theorem 8. If the neutrices N_{a+} and N_{b-} contain together with the function ν , the function $\lambda\nu$, where λ is a real number, and if φ is integrable from N_{a+} to N_{b-} then for each real number α the function $\alpha\varphi$ is integrable from N_{a+} to N_{b-} and holds:

$$(7)\qquad \int_{N_{a+}}^{N_{b-}} (\alpha\varphi)(x)\,dx = \alpha\int_{N_{a+}}^{N_{b-}} \varphi(x)\,dx$$

Theorem 9. If the function φ and ψ defined on $]a,b[$ are integrable from N_{a+} to b, then the function $\varphi+\psi$ is also integrable from N_{a+} to b, and the following holds:

$$(8)\qquad \int_{N_{a+}}^{b} (\varphi+\psi)(x)\,dx = \int_{N_{a+}}^{b} \varphi(x)dx + \int_{N_{a+}}^{b} \psi(x)dx$$

In similar hypotheses the following holds:

$$(8')\qquad \int_{a}^{N_{b-}} (\varphi+\psi)(x)dx = \int_{a}^{N_{b-}} \varphi(x)dx + \int_{a}^{N_{b-}} \psi(x)dx$$

Theorem lo. If the functions φ and ψ, defined on $]a,b[$ are integrable from N_{a+} to N_{b-} , then the function $\varphi+\psi$ is also integrable from N_{a+} to N_{b-} and the following holds:

$$(9)\qquad \int_{N_{a+}}^{N_{b-}} (\varphi+\psi)(x)\,dx = \int_{N_{a+}}^{N_{b-}} \varphi(x)dx + \int_{N_{a+}}^{N_{b-}} \psi(x)\,dx$$

Let u and v be two real functions, defined on $]a,b[$, derivable on this interval, and let N_{a+} be the neutrix introduced ot the beginning of this paragraph.

Theorem 11. If

1. for each $\xi \in]a,b[$, u' and v' are integrable from ξ to b;

2. $\lim_{\xi \to b} u(\xi)\, v(\xi)$ exists and is finite

3. the function $u(x)v(x)$ is neutralised by N_{a+};

4. one of the functions uv' und $u'v$ is integrable from N_{a+} to b;

then the other of the functions appearing at 4 is also integrable from N_{a+} to b, and holds:

$$(10) \qquad \int_{N_{a+}}^{b} u(x)v'(x)\,dx = \left[u(x)v(x)\right]_{N_{a+}}^{b} - \int_{N_{a+}}^{b} u'(x)v(x)\,dx$$

where $$\left[u(x)\, v(x)\right]_{N_{a+}}^{b} = \lim_{x \to b} u(x)v(x) - (u\, v)(N_{a+})$$

If N_{b-} is the neutrix introduced at the beginning of this paragraph, and the hypotheses of theorem 11 with respect to the two integration limits change their place, then holds:

$$(10') \int_{a}^{N_{b-}} u(x)v'(x)dx = \left[u(x)v(x)\right]_{a}^{N_{b-}} - \int_{a}^{N_{b-}} u'(x)v(x)\,dx$$

Mainteining the meanings of the notations N_{a+} and N_{b-} the following theorem takes place:

Theorem 12. If

1. for each $\xi, \eta \in]a,b[$, u' and v' are integrable functions from ξ to η;

2. the function $u(\xi)v(\xi)$ is neutralised by N_{a+}

3. the function $u(\eta)v(\eta)$ is neutralised by N_{b-}

4. one of the functions uv' and $u'v$ is integrable from N_{a+} to N_{b-};

then the other of the functions appearing at 4 is also integrable from N_{a+} to N_{b-} and the following holds:

$$(12) \qquad \int_{N_{a+}}^{N_{b-}} u(x)v'(x)dx = \left[u(x)v(x)\right]_{N_{a+}}^{N_{b-}} - \int_{N_{a+}}^{N_{b-}} u'(x)v(x)\,dx$$

Further on we shall establish the formula of iterate integration by parts for neutralised integrals

Let u,v be two functions defined on $]a,b[$, n time derivable on this interval

Theorem 13. If

1. for each $\xi \in]a,b[$,the function $u^{(n)}$ and $v^{(n)}$ are integrable from ξ to b,

2. the limit $\lim_{x\to b} u^{(k)}(x)v^{(n-k-1)}(x)$, $k=0,1,2,\ldots,n-1$ exists and is finite,

3. the functions $u^{(k)}(\xi)v^{(n-k-1)}(\xi)$, $k= o,1,2,\ldots,n-1$ are neutralised by the neutrix N_{a+} ,

4. one of the functions $u(x)v^{(n)}(x)$ and $u^{(n)}(x)v(x)$ is integrable from N_{a+} to b,

then the other of the functions appearing at 4 is also integrable from N_{a+} to b and the following holds:

$$(13)\quad \int_{N_{a+}}^{b} u(x)v^{(n)}(x)dx = \sum_{k=o}^{n-1}(-1)^k\left[u^{(k)}(x)v^{(n-k-1)}(x)\right]_{N_{a+}}^{b} + (-1)^n \int_{N_{a+}}^{b} u^{(n)}(x)v(x)dx$$

As the iterated formula (13) ,we can deduce the corresponding iterated formulae for the relations(11')and(12).

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UTILIZAREA CALCULULUI CU NEUTRICI PENTRU INCLUDEREA UNOR CLASE DE FUNCTII CARE NU SINT LOCAL INTEGRABILE IN SPATIUL DISTRIBUTIILOR

Rezumat

In această primă parte a lucrării se stabilesc regulile calculului cu neutrici ce vor fi efectiv folosite în paragrafele următoare.

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ASPECTS CONCERNAND L'EFFET HELIO-THERMAL. APPLICATION AU STOCAGE DE L'ENERGIE SOLAIRE

De Sabata C., Crăciun P., Mareu C.

1. Introduction. La conservation de la chaleur obtenue par la thérmoconversion de l'énergie solaire pose le problème des basins de stocage (en terre, en pierre, en eau) et l'une des idées proposées concerne l'utilisation des lacs, à eau douce ou salée, d'où la chaleur peut être récupérée au fur et au mesure qu'elle est necessaire.

Etant donné que la profondeur d'un basin de stocage est un élément important dans le calcule et que la surface étandue de celui-ci entraine une évaporation forte nous nous sommes proposés de realiser un étude théorique et expérimental concernand les basins à eau douce et à eau salée.

2. Basin à eau douce. Soit un basin à eau douce, trés large, d'où on sépare un élément dV à la surface S et profondeur dy (fig.1). A y = 0 l'insolation est constante, égale à I_o $[W/m^2]$, la température ambiante est constante T_o et l'ensamble se trouve dans un régime stationnaire. Soit ρ $[kg/m^3]$ la densitée constante de l'eau. L'aténuation de l'intesité avec la profondeur serra [1,2]:

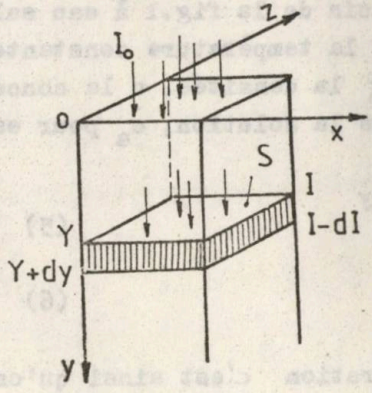

Fig.1. Modèle de l'élement de volume soumis à l'insolation

$$I = I_o e^{-\mu y} \qquad \mu = ct \qquad (1)$$

L'énergie absorbée par le volum élémentaire dV dans un temps t

échanffera l'eau conforme au

$$\left(\frac{dI}{dy}\right).S.dy.t = \rho.S.dy.c_a \Delta T_y \qquad (2)$$

$$\Delta T_y = (T_f - T_i)_y$$

et en résulte :

$$\Delta T_y = \frac{\mu}{\rho} \frac{I_o}{c_a} .t\, e^{-\mu y} \qquad (3)$$

$$T_{fy} = T_{iy} + \frac{\mu}{\rho} . \frac{I_o}{c_a} t\, e^{-\mu y} \qquad (3')$$

En admétand qu'au moment initial le tube élémentaire avait la même température sur toute sa longuer on peut écrire :

$$T_{fy} = T_{io} + \frac{\mu}{\rho} \frac{I_o}{c_a} t.e^{-\mu y} \qquad (3'')$$

Conclusion : Dans les basin à eau douce la température decroît d'aprés une vòie exponentielle avec la profondeur ayant la valeur maximum à la surface :

$$T_{fo} = T_{io} + \frac{\mu}{\rho} \frac{I_o}{c_a} .t \qquad (3'''')$$

Dans des conditions données $\frac{\mu}{\rho} . \frac{I_o}{c_a} = k$ (const) et

$$T_{fy} = T_{io} + k\, e^{-\mu y} \qquad (4)$$

3. Basin à eau salée. Soit le basin de la fig.1 à eau salée ($ClNa+H_2O$) solution saturée ; on admet la tempèrature constante sur toute la longuer et on notte par ρ la densitée, c la concentration et c^m la chaleure specifique de la solution, c_a pour eau c_s pour ClNa. Soit

$$\rho = \rho_o e^{ay} \qquad c = c_o e^{ay} \qquad (5)$$

$$c^m = c_a\left[1 - c(1 - \frac{c_s}{c_a})\right] \qquad (6)$$

L'atténuation en dépandand de la concentration c'est ainsi qu'on peut écrire $\mu = \beta c$; avec $a \cong 10^{-5}/m$, on écrit :

$$c \cong c_o(1 + ay) \qquad (5')$$

et la rel.(3) devienne :

$$\Delta T_y = \frac{\beta c_o e^{ay}}{\rho_o e^{ay}} \cdot \frac{I_o . t\, e^{-\beta c y}}{c_a\left[1 - c(1 - \frac{c_s}{c_a})\right]} \qquad (6')$$

et avec

$$\frac{1}{1 - c(1 - \frac{c_s}{c_a})} \cong 1 + c(1 - \frac{c_s}{c_a}) \qquad (7)$$

en utilizand (5') et (7) la rel.(6) devienne :

$$\Delta T_y = \frac{\beta c_o}{\rho_o} \frac{I_o . t}{c_a} \left[1 + c_o(1+ay)(1 - \frac{c_s}{c_a})\right] e^{-\beta c_o(1+ay)y} \qquad (8)$$

on notte : $1+ay=z$ et $A = \frac{\beta c_o}{\rho_o} \frac{I_o}{c_a} t$; $\beta = c_o(1 - \frac{c_s}{c_a})$; $N = \frac{\beta c_o}{a}$ (9)

et (8) devienne :

$$\Delta T_y = A\left[1 + \beta . z\right] e^{-N(z-1)z} \qquad (10)$$

on fait :

$$\frac{d}{dz}(\Delta T_y) = 0 \qquad (11)$$

at avec $z \neq \infty$ on obtenne :

$$2ABNz^2 - (ABN - 2AN)z - (AN + AB) = 0 \qquad (12)$$

En utilisant les notations $P = 2ABN$; $Q = AN + AB$ et $R = ABN + 2AN$ (13)

en resulte :

$$P z^2 - Rz - Q = 0 \qquad (14)$$

$$z_{1,2} = \begin{cases} \dfrac{R + \sqrt{R^2 + 4PQ}}{2P} & (+) \\[2ex] \dfrac{R - \sqrt{R^2 + 4PQ}}{2P} & (-) \end{cases} \qquad (15)$$

la solution positive étant la seule accebtable du point de vue physique. La position du maximum serra :

$$T_{fy} = T_{io} + A\left[1 + B(1+ay)\right] e^{-\beta c_o(1+ay)y} \qquad (16)$$

Dans la fig.2 on a donné T(y) pour (4) et (16).

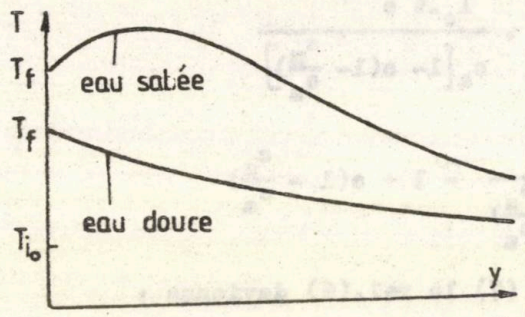

Fig.2 Alure théorique de T=f(y)

4. Essais éxpérimentals. On a utilisé comme élément d'essai un tuyau en PVC avec Ø = 10,2 cm; h = 132 cm. chauffé à la partie superieur à l'aide d'ume lampe Phylips de 400 W munite or non d'un filtre de i.r. On a fait les mesurages en régime stationnaire et les resultats sont donnés dans la fig.3 pour le tuyan rempli à l'eau douce, sallée, avec filtre de i.r.et sans filtre.

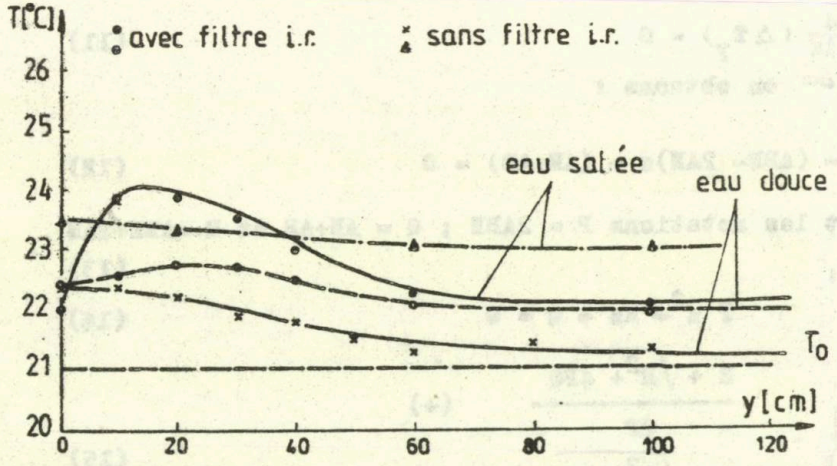

Fig.3 T=f(y) resultats expérimentaux

L'aténuation de la radiation en dependend directement de la concentration et de la longueur d'onde de la radiation électromagnétique, les quatre courbes mettent en évidence ces particularités.

5. Conclusions . L'étude presenté montre les suivants aspects :

5.1. L'effet héliothermal est fort dans les basins à eau sallée pour un spectre comparable au spectre solaire.

5.2. L'échauffement de la solution ne se produit pas de la base vers la partie supérieure comme on admet en général [4 , 5], mais de la surface vers le fond, en passant par un maximum entre 20-30 cm.

5.3. L'efficience maximum du stocage est obtenue par des basins pas profonds avec une large surface, à condition qu'ils soient couverts pendent les périodes non insoleillées avec une feuille en plastique, des biles flottantes, or d'autre système.

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ASPECTE PRIVIND EFECTUL HELIO-TERMAL, APLICATII IN STOCAJUL ENERGIEI SOLARE

Rezumat

O notată eficienţă de stocaj a energiei solare constă în utilizarea efectului heliotermal. In lucrare se concepe un mecanism pe baza căruia se explică teoretic acest efect. In ultima parte a lucrării se verifică experimental relaţiile teoretice obţinute.

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L'ABSORPTION DE LA RADIATION ÉLECTROMAGNÉTIQUE PAR UN FERROFLUIDE À BASE DE Fe_3O_4

De Sabata C., Marcu C., Miron I., Luminosu I.

1. Introduction. L'idée de l'absorption directe de la radiation solaire par un fluide noir a été expérimentée par Minardi et Chunag [1] et étudiée théoriquement par M.Abdel Rahman, P.Fumeaux et P.Suter [2]. Dans ces ouvrages les auteurs utilisent comme matériel absorbant le carbone sous la forme de graphite. Dans un ouvrage publié antérieurement [3] nous avons montré que les ferrofluides utilisés comme absorbants de l'énergie solaire présenteraient aussi l'avantage d'un entraînement simple par des méthodes magnétiques spécifique au ferrofluide.

2. Préparation du ferrofluide . Le ferrofluide soumis à l'étude optique a été préparé par une méthode mécanique de dispersion [4], de la magnétite industrielle avec un mélange d'acide oléique et d'huile pour les transformateurs électriques, (ρ_0 = = 0,0875 g/cm^3). Après quelques centaines d'heures de mouture, le liquide obtenu a été soumis à une décantation magnétique en obtenant un ferrofluide dilué de densité ρ_1 = 0,8975 g/cm^3 . Ensuite, ce ferrofluide a été concentré par l'introduction de la magnétite synthétique, en différentes proportions, avec l'acide oléique nécessaire dans un moulin à billes où il a été mélangé pendent quelques centaines d'heures. On a obtenu des échantillons ayant les densités dans l'intervalle 0,8875 - 1,145 g/cm^3.

3. Résultats expérimentaux et interprétations

Le ferrofluide ayant les densités indiquées ci-dessus a été soumis à l'étude de l'absorption dans le domaine des longueurs d'onde du visible et infrarouge.

Dans la fig.1 on présente l'absorption de la radiation électromagnétique dans le visible par le ferrofluide ayant des dif-

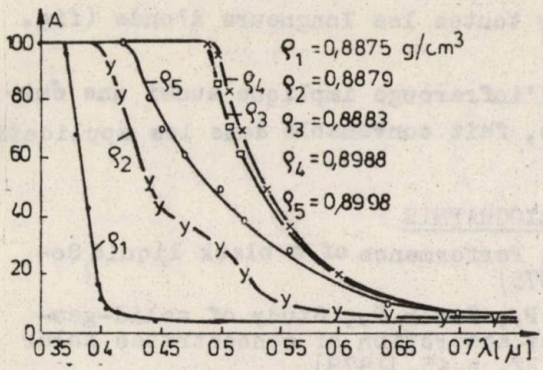

Fig.1. L'absorption du ferrofluide ayant des densité différentes

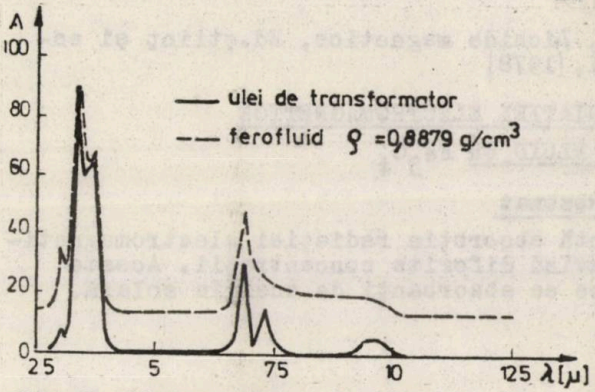

Fig.2. L'absorption de l'huile de transformateur et du ferrofluide ayant la plus petite concentration

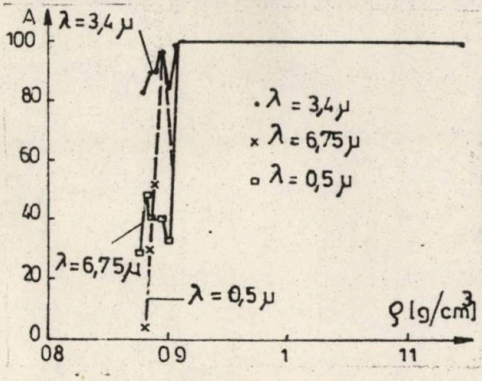

Fig.3. L'absorption de quelques raies spectrales par le ferrofluide en fonction de la densité

férentes densités. On spécifie que pour les densités plus élevées que celles présentées dans la figure l'absorption est totale.

Dans la fig.2 est présentée l'absorption de la radiation électromagnétique dans l'infrarouge pour l'huile de transformateurs lequel constitue la base du ferrofluide et pour le ferrofluide ayant la plus petite concentration.

Dans la fig.3 est présentée l'absorption de quelques raies spectrales par le ferrofluide en fonction de la densité.

L'analyse des courbes présentées ci-dessus met en évidence:

1. L'huile de transformateur a l'absorption négligeable dans les domaines visible et infrarouge et totale dans l'ultraviolet.
2. L'introduction d'une quantité réduite de magnétite augmente à 100 % le facteur d'absorption pour les longueurs d'onde petites mais modifie peu le domaine dans le rouge et l'infrarouge ; l'agrandissement de la concentration amène le facteur d'absorption à 100 % sur toute la largeur du spectre étudié (fig.1).

3. La magnétite renforce la caractéristique spectrale du milieu dispersant quasiégalement sur toutes les longueurs d'onde (fig. 2) dans le domaine visible.

4. L'absorption faible dans l'infrarouge implique aussi une émission faible dans ce domenine, fait convenable dans les applications solaire.

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ABSORBTIA RADIATIEI ELECTROMAGNETICE INTR-UN FLUID CU Fe_3O_4

Rezumat

In lucrare se prezintă absorbţia radiaţiei electromagnetice de către un ferofluid avînd diferite concentraţii. Aceste ferofluide pot fi utilizate ca absorbanţi de energie solară.

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PROPRIETATI OPTICE ALE CUPOLEI DE STICLA UTILIZATA IN CONSTRUCTIA UNUI PIRANOMETRU

De Sabata C., Mihalca I., Marcu C., Damian I., Ercuţa A.

Introducere. Pentru măsurarea intensităţii radiaţiei solare se utilizează în majoritatea cazurilor receptori integrali protejaţi prin cupole emisferice de sticlă. In acest scop se utilizează în general o mare varietate de materiale optice [1].

Lucrarea examinează influenţa cupolei emisferice de sticlă asupra iluminării produse în planul receptor, determinînd forma şi mărimea suprafeţei caustice.

Date experimentale. Trecerea luminii prin cupola sferică este însoţită de micşorarea fluxului datorită pierderilor prin reflexie şi absorbţiei în material. Contribuţia celor două fenomene depinde de natura materialului, de indicele de refracţie şi de starea suprafeţei. Pentru unghiuri de incidenţă $i \leq 60^{\circ}$ putem admite că pierderile prin reflexie sînt constante. In domeniul spectral 0,4-1,1 μm. Sticla utilizată prezintă un coeficient de transmisie $\simeq$ 94 % (fig.1).

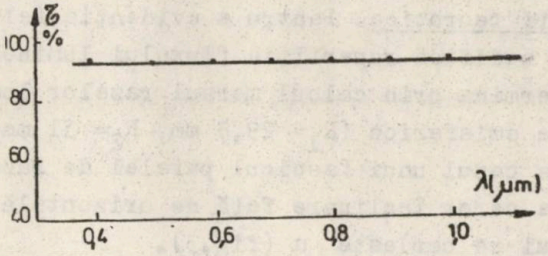

Fig.1 Transparenţa cupolei de sticlă

Iluminarea suprafeţei receptoare situaţia în planul xoy (fig.2 a) este uniformă în regiunile în care razele emergente din

cupolă întîlnesc direct această suprafaţă şi creşte în regiunea în care o parte din aceste raze suferă reflexii pe faţa interioară a cupolei, Ca rezultat apare o suprafaţă caustică a cărei formă şi mărime depinde de înălţimea Soarelui pe cer (fig.2 a) Lărgimea maximă (Δy_{max}) a causticii se obţine în cazul cînd Soarele este la orizont iar pentru înălţimi $\simeq 60^{\circ}$, $\Delta y \to 0$ (fig.2 b). Deplasarea aparentă a Soarelui pe cer determină o rotaţie a suprafeţei caustice în jurul centrului O şi totodată o micşorare a acesteia.

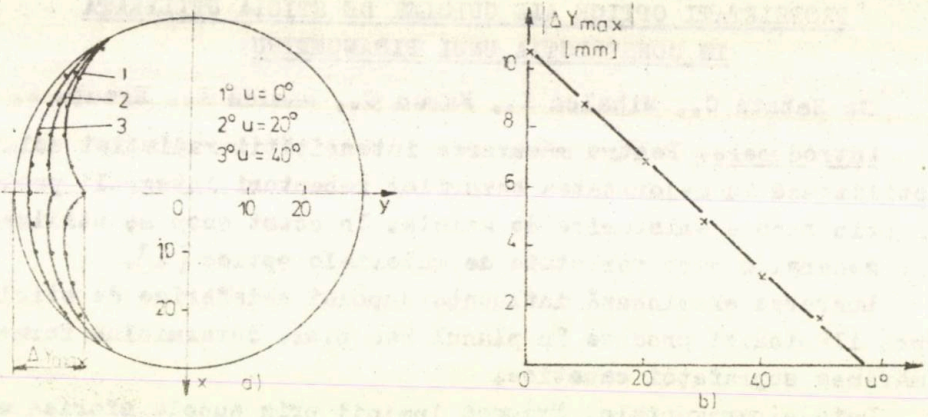

Fig.2 Forma şi mărimea suprafeţei caustice

Cunoscînd dimensiuniile suprafeţei caustice vom dimensiona astfel suprafaţa circulară receptoare încît raza acesteia să difere de raza cupolei cu mărimea Δy_{max} corespunzător unghiului u = 0. In acest mod sînt evitate erorile de măsurare introduse prin iluminarea succesivă (datorită rotaţiei causticii) a regiunilor albe, respectiv negre ale suprafeţei receptoare.

Consideraţii teoretice. Pentru a evidenţia felul în care cupola de sticlă modifică repartiţia fluxului luminos din planul receptor vom determina prin calcul mersul razelor luminoase în cazul unei cupole emisferice (R_1= 29,5 mm, R_2= 31 mm, n = 1,6).

Vom examina cazul unui fascicul paralel de raze conţinute în plane $x = a_i$ a căror înclinare faţă de orizontală (egală cu înălţimea Soarelui pe cer)este u (fig.3).

Fie un punct de incidenţă arbitrar M_1 caracterizat prin parametrii (a_i, $r_i \cos \varepsilon$, $r_i \sin \varepsilon$).

Ecuaţia razei incidente în punctul M_1, de vector director $\vec{v}$(0, cos u, sin u) este :

$$x - x_1 = 0 \quad ; \quad \frac{y - y_1}{\cos u} = \frac{z - z_1}{\sin u} \qquad (1)$$

Dacă normala în punctul de incidenţă are vectorul director $\vec{v}(a_1/R_2 \ , \ r\cos\varepsilon/R_2, \ r\sin\varepsilon/R_2)$, atunci unghiul de incidenţă se calculează din relaţia :

$$\cos i = \left[1 - \left(\frac{a}{R_2}\right)^2\right]^{\frac{1}{2}} \cos(\varepsilon - u) \qquad (2)$$

Relaţia (2) arată că pentru o înclinare dată ($u = ct$) unghiul de incidenţă pe suprafaţa sferică se modifică odată cu poziţia punctului M_1. Pentru $\varepsilon = u$ incidenta devine normală. Dacă $\varepsilon > u$, unghiul de incidenţă i creşte şi odată cu el creşte unghiul de incidenţă i' (pe suprafaţa sferică de rază R_1) pînă la atingerea valorii limită $i = ie$; razele pentru care $i > ie$ nu mai contribuie la iluminarea planului receptor. Razele ce ajung pe suprafaţa receptoare direct după reflexiile succesive în punctele M_1 şi M_2 contribuie la iluminarea uniformă a suprafeţei, deoarece deviaţia lor faţă de direcţia iniţială este neglijabilă.

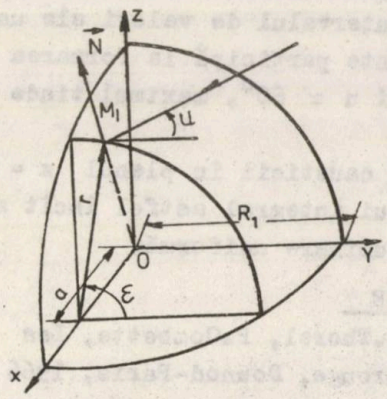

Fig.3 Parametrii unei raze incidente

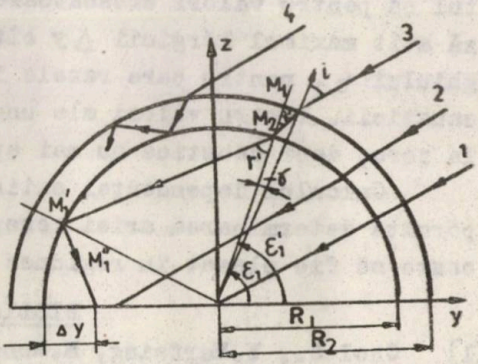

Fig.4 Mersul razelor incidente în planul xoz

Razele ce suferă reflexii pe faţa interioară contribuie la creşterea iluminării şi la apariţia causticii (fig.4).

Determinarea coordonatelor punctului M_1 conduce la expresia lărgimii Δy a causticii :

$$\Delta y = R_1 \left[\frac{\sin(\varepsilon_1 + \gamma - 2r')}{\operatorname{tg}(\varepsilon_1 + \gamma - 3r')} - \cos(\varepsilon_1 + \gamma - 2r') - 1 \right]$$

Variaţia mărimii Δy ca funcţie de unghiul ε_1 pentru valori

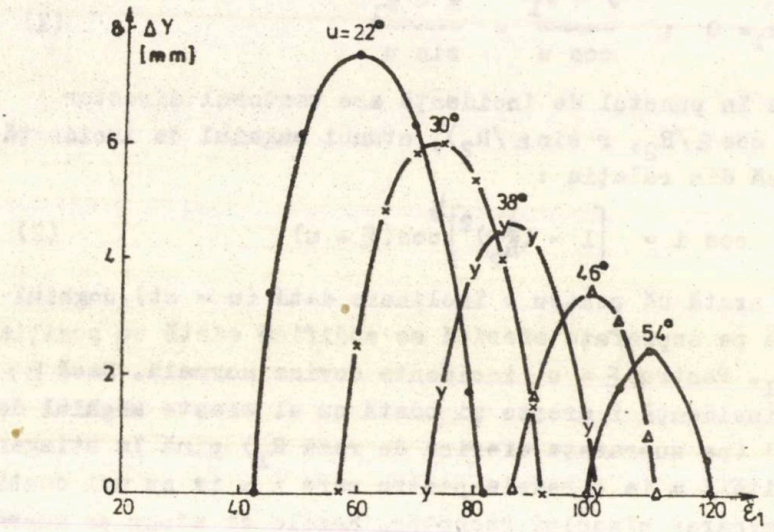

Fig.5 Dependenţa lărgirii causticii de parametrii ε şi u

ale parametrului "u" este prezentată în fig.5. Se remarcă faptul că pentru valori crescătoare ale parametrului u se micşorează atît maximul lărgimii Δy cît şi intervalul de valori ale unghiului ε_1 pentru care razele incidente participă la formarea causticii. Pentru valori ale unghiului u $\simeq 60^{\circ}$, maximul tinde la zero, deci caustica nu mai apare.

Calculul dependenţei extinderii causticii în planul z = 0 permite determinarea ariei receptorului integral astfel încît acesta să fie plasat în regiunea cu iluminare uniformă.

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OPTICAL PROPETIES OF GLASS CUPOLA USED IN A PYRRANOMETER CONSTRUCTION

Abstract

The work examines the influence of a hemispherical glass cupola upon the receiving plane lighting, finding the shape and the extention of nonuniformely lightend area.

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ASUPRA DEBITULUI ENERGETIC LA UN HELIOCAPTATOR PLAN FURNIZOR DE AER CALD TEHNOLOGIC

De Sabata C., Marcu C., Luminosu I.

1. Introducere. In vederea evaluării performanţelor unui heliocaptator plan pentru producerea de aer cald [1] este necesar un studiu asupra dependenţei parametrice a puterii utile ca funcţie de debitul agentului caloportor, de lungimea de parcurs a acestuia, precum şi de insolaţie.

Debitul energetic al unui astfel de heliocaptator plan care funcţionează în circuit deschis se calculează cu relaţia :

$$P = \dot{m}\, c_p t \qquad (1)$$

Supra temperatura (t), dependentă de intensitatea radiaţiei solare (E) şi de lungimea tubului de curent (x) [2, 3], determină corelarea implicită a debitului energetic furnizat de heliocaptator şi mărimile mai sus precizate.

Prezenţa lucrare urmăreşte stabilirea performanţelor unui heliocaptator plan [1] din punctul de vedere al debitului energetic furnizat, prin corelarea acestuia cu debitul fluidului caloportor, lungimea tubului de curent şi valoarea intensităţii radiaţiei solare.

2. Rezultate experimentale şi interpretări. Măsurătorile au fost efectuate în condiţiile echinocţiului de primăvară cu debite variabile de agent caloportor şi pentru 10 valori ale intensităţii radiaţiei solare.

Introducînd în (1) expresia supratemperaturii se obţine :

$$P = \frac{\dot{m}\, c_p E \overline{DA}\, L}{K L_c} \left[1 - \exp\left(- \frac{Ka}{\dot{m}\, c_p} x\right) \right] \qquad (2)$$

In fig.1 este prezentată dependența de lungimea de parcurs a debitului energetic furnizat de heliocenvertorul plan pentru E = 940 W/m^2. Intrucît măsurătorile la această valoare a intensității radiației solare au fost efectuate la ora 13 (ora oficială), în relația (2) s-a luat $\frac{L}{L_o} \simeq 1$.

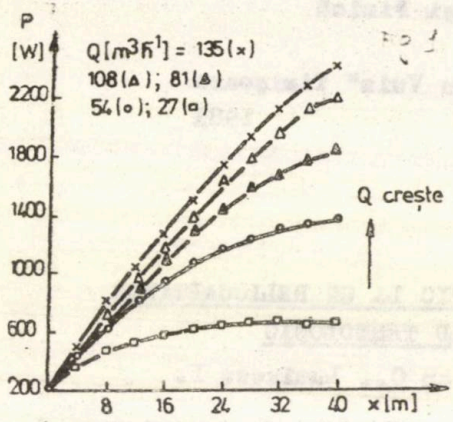

Fig.1 Dependența debitului energetic de lungimea de parcurs a agentului caloportor

Din fig.1 rezultă că debitele energetice extrase (P) mari se obțin la debite mari ale agentului caloportor, fapt care se explică prin îmbunătățirea schimbului de căldură între placa absorbantă și agent. Intrucît pentru modulul studiat palierul lui P extras se atinge numai pentru debite mici ale agentului, rezultă existența unor rezerve ce pot fi valorificate prin înserierea mai multor astfel de module. Această constatare se evidențiază prin analiza evoluției expresiei

$$\text{grad } P = E\,\overline{DA} \cdot \exp\left(-\frac{Ka}{\dot{m}\, c_p}\, x\right) \qquad (3)$$

ilustrată în fig.2.

Evident că mărimea lungimii de parcurs nu trebuie să depășească o anumită limită impusă de costul instalației modulare, căci grad $P \to 0$ pentru $x \to \infty$.

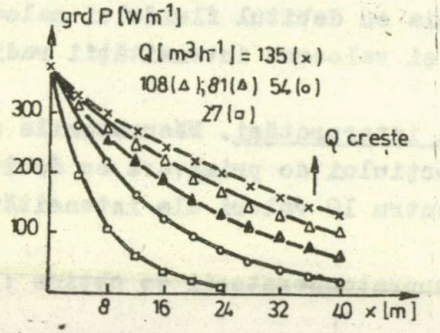

Fig.2 Dependența gradientului debitului energetic de lungimea de parcurs a agentului caloportor

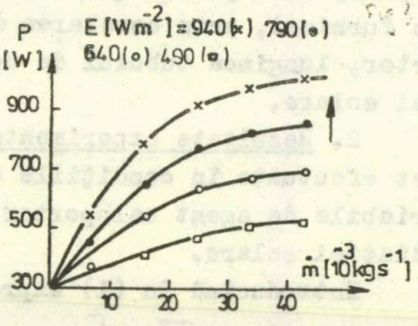

Fig.3 Dependența debitului energetic de debitul agentului caloportor

Dependenţa explicită a debitului energetic extras P, de debitul $\dot{m}$, ilustrată în fig.3, evidenţiază, de asemeni, avantajele utilizării unor valori ale lui $\dot{m}$ în special în condiţiile unor insolaţii bune. Avînd în vedere însă, faptul că sporirea lui $\dot{m}$ determină o creştere corespunzătoare a consumului de energie la turbina de antrenare a agentului este necesară şi o analiză a variaţiei pantei puterii ca funcţie de debit :

$$p = \frac{dP}{d\dot{m}} = c_p \frac{E\,\overline{DA}}{K}\left[1 - \left(1 + \frac{KaL}{\dot{m}c_p}\right)\exp\left(-\frac{KaL}{\dot{m}c_p}\right)\right] \qquad (4)$$

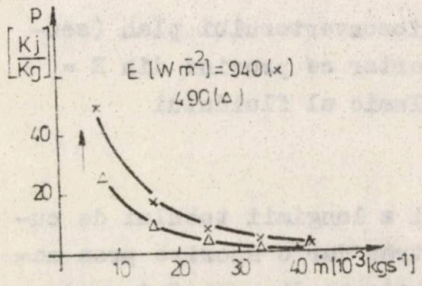

Fig.4 Dependenţa pantei debitului energetic de debitul agentului caloportor

în care L reprezintă lungimea totală de parcurs pentru un singur model. Dependenţa prezentată în (4) este ilustrată grafic, pentru două valori ale insolaţiei în fig.4.

Creşterea debitului agentului caloportor peste o anumită valoare este neavantajoasă şi datorită faptului că în condiţiile unei instalaţii mai slabe (de exemplu la orele 9 şi 15) sporirea debitului nu conduce la creşteri seminficative a energiei furnizate de heliocconvertor fig.5.

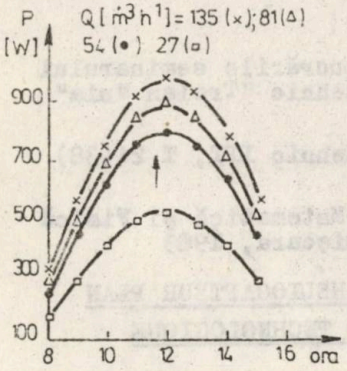

Fig.5 Variaţia orară a debitului energetic

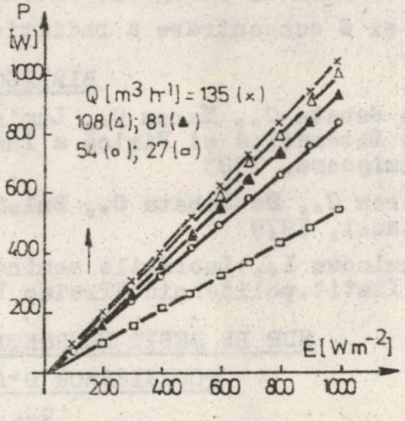

Fig.6 Dependenţa debitului energetic de intensitatea radiaţiei solare

Această observaţie rezultă mai clar din fig.6 în care s-a reprezentat variaţia debitului energetic extras ca funcţie de intensitatea radiaţiei solare pentru diverse debite ale agentului caloportor.

Pentru o valoare dată a debitului agentului caloportor, relaţia (2) poate fi liniarizată în forma

$$P = E \cdot C \qquad (5)$$

în care

$$C = \frac{\dot{m}\, c_p\, \overline{DA}\, L}{K\, L_c}\left[1 - \exp\left(-\frac{K \cdot a \cdot L}{\dot{m}\, c_p}\right)\right]$$

indică caracteristica de putere a helioconvertorului plan (semnifică puterea furnizată de helioconvertor ce provine din $E = 1\left[W/m^2\right]$). Pe figuri s-a dat debitul volumic al fluidului $Q\left[m^3h^{-1}\right]$.

3. Concluzii :

a) Mărirea debitului de fluid şi a lungimii tubului de curent favorizează debitul energetic extras dar o sporire prea mare a celor doi parametri nu este avantajoasă din punct de vedere economic.

b) Valorile optime ale debitului de flùid şi ale lungimii tubului de curent trebuie stabilite pe baza diagramelor prezentate în lucrare în corelare cu procesele tehnologice concrete care utilizează agentul caloportor.

c) Debitul energetic extras crescînd nelimitat cu E apare utilă şi o concentrare a radiaţiei pe captator.

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SUR LE DEBIT ENERGETIQUE A UN HELIOCAPTEUR PLAN FOURNISSEUR D'AIRE CHAUD TECHNOLOGIQUE

Résumé

Dans l'ouvrage on présente l'étude de la dépendence paramétrique du débit énergétique utile fourni par un heliocapteur plan pour l'aire chaud technologique.

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THE DEPENDENCE OF THE STRUCTURE OF Fe-Zn ELECTROLITICAL THIN LAYERS ON THE OBTAINING CONDITIONS

De Sabata C., Marcu C., Damian I., Palade T., Farbaş N.*

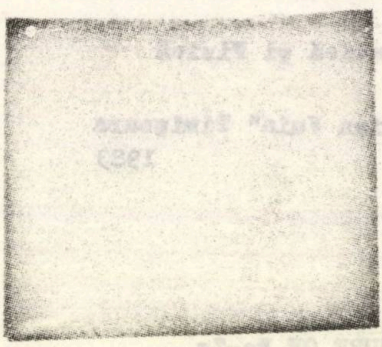
Introduction. There are few elements well soluble in iron, so that makes more interesting the study of ferromagnetic properties of iron-zinc alloy.

The obtaining from melting of Fe-Zn alloy, having anticorosive properties, involves great consumption of energy and materials [1]. The electrolytical obtaining of this alloy is much more economical and easier. More over, the structure, the composition and the properties of alloy may be controlled by varying the deposition conditions.

The purpose of the present paper is to establish the influence of the electrolytical current density on the structure of the alloy.

2. Experimental results and interpretation. Fe-Zn electrolytical layers were obtained on copper sheet as described in [2]. The samples were metallograficaly analysed after chemical treatement by 5 % HCL/C_2H_5OH solution, that blackened the Zn only . Fig.1 (a,b,c,d and e) showsthe photographs of the samples corresponding to the current density values of 1,2,3,4 and 5 A/dm^2, respectively : zinc zones appear dark. One can observe an increasing of Fe amount by increasing of current density up to 5 A/dm^2, after wich the proportion of Zn increases again.

One can also observe the tendency in homogenizing of material up to 4 A/dm^2 (including). So, while at 1 A/dm^2 current den-



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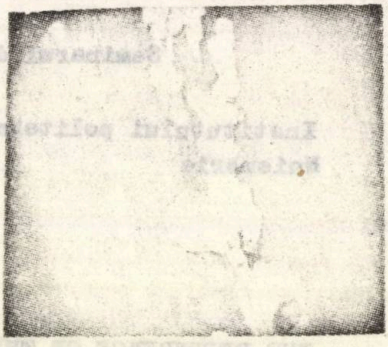

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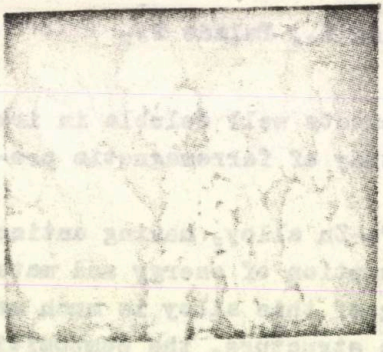

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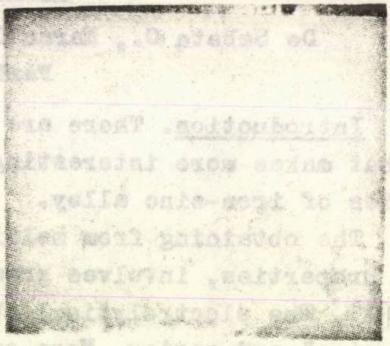
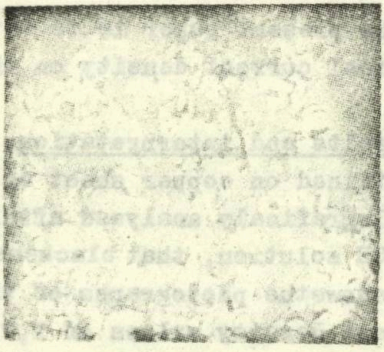


d



e

Fig.1 Metallografical structure of Fe-Zn electrodepozited alloy ; a - 1 A/dm^2 ; b - 2 A/dm^2 ;c - 3 A/dm^2 ; d - 4 A/dm^2 ; e - 5 A/dm^2 .

sity the zinc crystallites, sharply outlined, are deposed in a fine iron matrix, at 2 A/dm^2 an iron lamellar structure occurs, the the surface of wich extends when current density increases up to 4 A/dm^2 , when the sample gets a strong homogenity.

X ray analysis of the samples confirms the above conclusions. Thus, the diffraction spectra corresponding to current density values of 1 and 2 A/dm^2 indicate the presence of iron and and zinc, separately, while for 3, 4 and 5 A/dm^2 additional crystallografic planes occurs, the hexagonal structure of zinc vanishes and some changes of iron lattice constants values can be observed.

In the following table the interplanar distances for X ray diffraction maxima are presented.

I A/dm^2	1	2	3	4	5
	2,42	2,38			
Interplanare	2,10	2,10	2,10	2,10	2,10
distances	1,095	1,090	1,095	1,100	1,099
	1,098	1,028			
	1,616	1,613	1,600	1,600	1,600
	1,377	1,373			
$d.10^{10}$	1,246	1,243			
m	1,047	1,047			
			2,18		2,01
			1,82	1,82	1,82
			1,287	1,286	1,287
			1,23	1,25	1,23
			1,188		1,17
			1,35	1,35	
			1,04		1,04
Structure	mecha-nical mixtu-re of Fe and Zn	mecha-nical mixtu-re of Fe and Zn	inter-meta-lic com-pound	inter-meta lic com-pound	inter-meta-lic com-pond

The evolution of the above mentioned structure is also confirmed by scanning electronic microscopy.

3. Conclusions.

The structure of Fe-Zn alloy, electroliticaly deposed, depends on the current density. For small current density values (1-2 A/dm^2) a microscopic heterogeneous alloy are obtained (mechanical mixture of iron and zinc) in wich the amount of iron increases as the current density does.

For current density values of 3, 4 and 5 A/dm^2 the alloy consist in Fe_3Zn_{10} intermetallic compuond.

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DEPENDENTA DE CONDIŢIILE DE OBTINERE A STRUCTURII PATURILOR SUBTIRI ELECTROLITICE DE Fe-Zn

Rezumat

Lucrarea prezintă dependenţa proprietăţilor structurale ale aliajului Fe-Zn obţinut sub formă de pături subţiri prin depunere electrolitică.

Compoziţia şi structura aliajului variază în funcţie de densitatea de curent în timpul depunerii. La densităţi de curent din intervalul 3-5 A/dm^2 aliajul se prezintă sub forma unui compus intermetalic de tipul Fe_3Zn_{10} .

Seminarul de Matematică şi Fizică
al
Institutului politehnic "Traian Vuia" Timişoara
Noiembrie 1983

UNTERSUCHUNGEN DER HERSTELLUNGSBEDINGUNGEN UND DER MAGNETISCHEN EIGENSCHAFTEN DER ELEKTROLYTISCH NIEDERGESCHLAGENEN Fe-Zn-LEGIERUNG

De Sabata C., Marcu C., Mihailovici D., Damian I.,
Boldan M., Gangăl M.

A. Einleitung. In der Technik ist die Fe-Zn-Legierung durch ihre antikorrosiven Eigenschaften bekannt und dadurch wird diese als Schutzüberzüge [1] , die 31 % Zn enthalten, verwendet.

Systematische Untersuchungen der magnetischen Eigenschaften der Fe-Zn Legierung die man durch Schmelzen [2] erzeugte, führen zu Ergebnisse die ein ausergewöhnliches Verlalten der Legierung aufweisen, und zwar in dem Sinne, dass das Vorhandensein der Zn-Atomen im Kristallgitter des Eisen zu einer Vergrösserung des mittleren magnetischen Moment des Eisens führt (höchstwahrscheinlich durch die Veränderung der Austauschintegrale).

In der vorliegenden Arbeit sind einige der untersuchten magnetische Eigenschaften der dünnen elektrolytisch niedergeschlagenen Fe-Zn-Schichten wiedergegeben.

B. Herstellungsbedingungen der Fe-Zn-Legierung. Die dünnen Fe-Zn-Schichten sind elektrolytisch abgeschieden worden, in dem man folgendes Bad verwendete :

$FeSO_4 \cdot 7\,H_2O$	55	g
$ZnSO_4 \cdot 7\,H_2O$	28	g
Askorbinsäure	0,2	g
Zitronensäure	0,5	g
H_3BO_3	30	g
H_2O	1000	cc
pH-Wert	2,5	
Temperature	32°C.	

Die Stromdichte befand sich zwischen 0,5 A/[illegible] und 8 A/dm^2.

Die Schichten wurden auf Kupferplatten von 0,1 mm Dicke aufgetragen. Da sowohl dass metallographysche Gefüge, als auch die magnetischen Eigenschaften von der Stromdichte abhängig sind, untersuchten wir den Einfluss der Stromdichte während der Elektrolyse auf diese charakteristiken. Die Schichtdicke ist durch Wiegen bestimmt worden.

C. Experimentelle Ergebnisse und ihre Deutungen. Die Messungen der magnetischen Grössen der dünnen Fe-Zn-Schichten sind quasistatisch bei 50 Hz mit Hilfe einer Anlage, die gleichzeitig die induzierte Spannung im Remagnetisierungsprozess und die entsprechende Hysteresiskurve widergibt, durchgeführt worden.

In der Abbildungen 1 a un 1 b sind die Hystereseschleifen der Proben, die man bei Stromdichten von 1 A/dm^2 und 5 A/dm^2 erhielt, dargestellt.

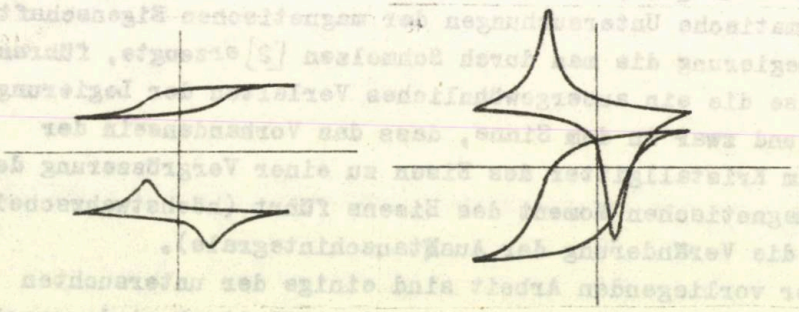

Abb.1 Hysteresiskurven der erhaltenen Fe-Zn-Schichten die bei $j = 1\ A/dm^2$ und $j = 5\ A/dm^2$

Die Analyse der Kurven aus Abb.1 ergibt, dass grosse Veränderungen sowohl in beizug auf den Sättigungszustand, als auch in bezug der Kerzitivkraft entstehen.

Diese Veränderungen sind in Abbildung 2 wieder gegeben, wobei M_{Sn}/M_{S1} das Verhältnis zwischen der Sättigungsmagnetisierung der Proben, die bei verchiedene Stromdichten erhalten sind, und der Sättigungsmagnetisierung jener Probe die bei der Kleinsten Stromdichte erzeugt wurde, darstellt.

Es wurde auch eine Strukturanalyse der Schichten durchgeführt, da die Koerzitivkraft vom Gefüge abhängig ist, während die Sättigungsmagnetisierung nicht.

In Abb.3 sind die Strukturen der dünnen Schichten, die man

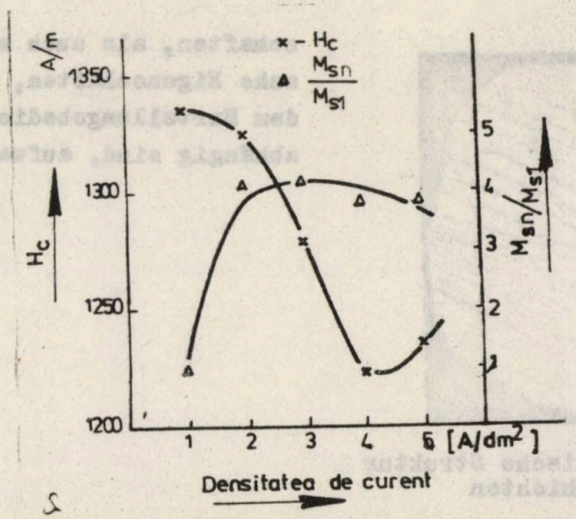

Abb.2 Veränderung der Koerzitivkraft und des Verhältnises M_{Sn}/M_{S1} in bezug von der Stromdichte.

Abb.3 Strukturen der erhaltenen Fe-Zn-Schichten bei Stromdichten j = 2 A/dm² und j = 4 A/dm².

bei einer Stromdichte von 2 A/dm² beziehungsweise bei 4 A/dm² erhielt, wiedergegeben.

In bezug der magnetischen Struktur ist zu erwähnen, dass die Auwesenheit der CROSSTIE-Wänden (Abb.4) den Ubergang von den Bloch-Wänden zu den Néel-Wänden andeuten.

D. Schlussfolgerungen. Die Fe-Zn-Legierung, die durch das beschriebene Verfahren erhalten wurde, erscheinen als festhaftende hellgraue Schichten, welche sowohl antikorrosive Eigen-

schaften, als auch magnetische Eigenschaften, die von dem Herstellungsbedingungen abhängig sind, aufweisen.

Abb.4 Magnetische Struktur der Fe-Zn-Schichten

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ASUPRA OBTINERII SI PROPRIETATILOR MAGNETICE ALE ALIAJULUI ELECTROLITIC DE Fe-Zn

Rezumat

In lucrare se prezintă procedeul de obţinere electrolitică a aliajului Fe-Zn precum şi dependenţa proprietăţilor structurale şi magnetice de densitatea de curent în timpul procesului de elaborare.

Seminarul de Matematică şi Fizică
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Noiembrie 1983

RECHERCHES EXPÉRIMENTALES SUR L'INFLUENCE DES AGENTS EXTÉRIEURS AU RENDEMENT D'UN CAPTEUR CYLINDRIQUE

De Sabata C., Mihailovici D., Baea R.,
Luminosu I.

Dans l'ouvrage on étudie l'influence des facteurs météorologiques sur les performances énergétiques d'un capteur cylindrique utilisé pour l'échauffement du bitume avec accent sur le rendement global de celui-ci. L'étude de l'influence des agents polluants sur le comportement d'un capteur plan [1] a montré que ceux-ci attaquent la surface du capteur en y déposant des composantes du milieu ambiant et qui entrent en réaction avec le matériau de sa surface. L'état de la surface des capteurs (plans ou cylindriques) influence le rendement optique et en même temps le rendament global (1) :

$$\eta_g = \eta_o \cdot \eta_{thérmique} \qquad (1)$$

Les mesurages expérimenteaux ont été efectués sur un capteur cylindrique avec du bitume, présenté dans [2] dont les caractéristiques principales étaient les suivantes :

$S_{activa} = 45.10^{-3}\ m^2$, la mase du bitume contenu par le réservoir $m_{bi} = 6,4$ kg ; chaleur spécifique du bitume $C_{bi} = 1672\ J/kg^oC$.

Le calcul du rendement global diurne a été fait en connaissant l'intensité du rayonnement solaire $E\left[W.m^{-2}\right]$ obtenue par les mesurages expérimenteaux avec un bolometre différentiel de type "Solaris" construit dans nôtre laboratoire.

Q - la chaleur utile diurne et donnée par la rélation :

$$Q = (m_{bi} C_{bi} + m_{rez} \cdot C_{rez}) \Delta t \qquad (2)$$

où Δt représente la différence entre la température maximale atteinte par le bitume dans le réservoir et celle ou commencement de mésurages.

Q_i - l'énergie solaire à la surface éxposée est calculée à l'aide de la courbe $E = f(\tau)$ (τ - l'interval du temps de mésurage. Le rendement global est calculé par :

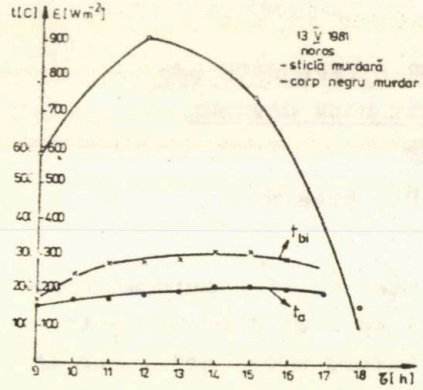


Fig.1 Variation de l'intensité du rayonement solaire dans le temps (la courbe 0) des températures du bitume (la courbe x) et du milieu ambiant (la courbe .)

$$\eta_g = \frac{Q_u}{Q_i} \qquad (3)$$

Les mesurages éxperineteaux ont été effectués dans les cas suivants :

1. Plaque de verre et plaque de l'absorbbeur sales (le dépôt naturel après sept mois de fonctionement octobre-mais 1981) (fig.1).

2. Plaque de verre et plaque métalique propres (lavés de l'eau) (fig.2, 3).

3. L'installation propre et couverte d'une feuille de polyestère.

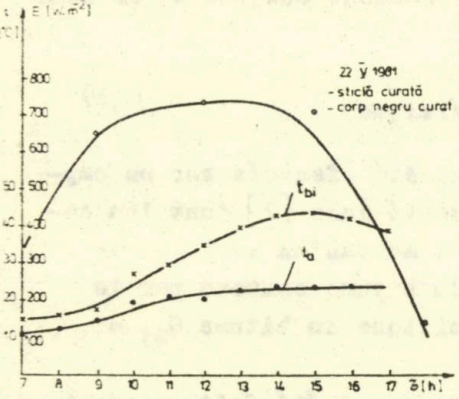


Fig.2 La dependence dans le temps de l'intensité du rayonnement solaire et des températures du bitume en 2-ème cas.

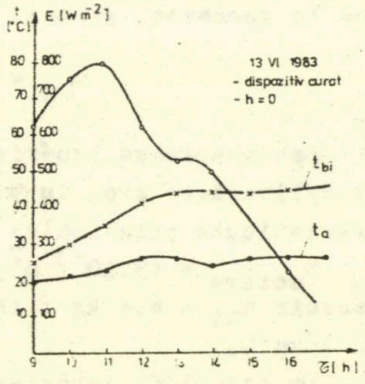
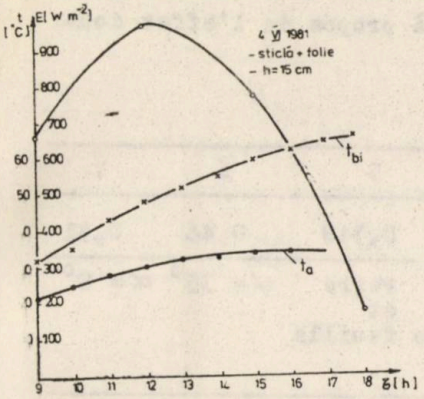


Fig.3 Variation de l'intensité du rayonnement solaire et des températures du bitume - l'installation horizontale



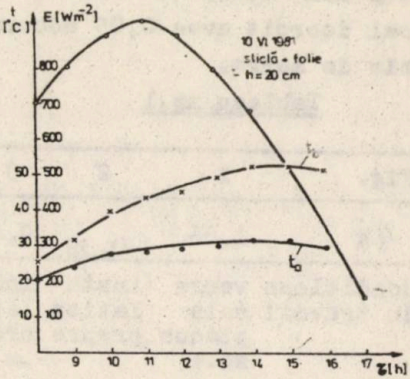
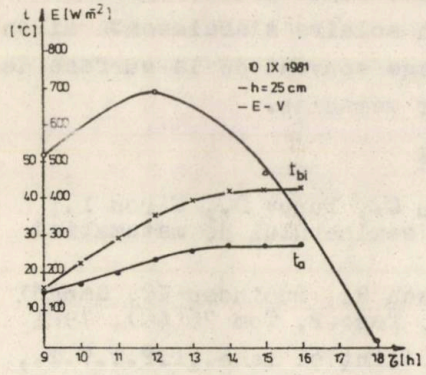


Fig.4,5 Variation de l'intensité du rayonnement solaire et des temperatures du bitume pour l'effet duble de serre

4. L'installation orientée vers le sud, $\alpha = 35^0$ (fig.6) (α - étant l'angle d'inclinaison du capteur par l'horisontale) et $\alpha = 8^0$(fig.7).



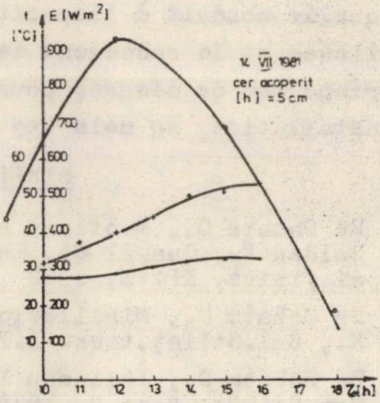


Fig.6 Variation de l'intensitée du rayonnement solaire et des températures du bitume - l'orientation de l'axe d'insollation Est - l'Quest

Fig.7 La dépendence dans le temps de l'intensité du rayonnement solaire et des températures du bitume - ciel couvert des mages.

Les randements calcule's par la relation (3) sènt presentés dans le tableau nr.1.

Les recherches éxpérimenteaux montrent que les dépôts polluants conduient à la diminution du rendement η jusqu'à la moitié de celui qui correspond à l'installation propre ;quand on útilise une feuille en polyestère, la durée effective du chauffage

augement aussi que la temérature finale mais le rendement global decroit avec 0,05 accord avec 3 à propos de l'effet double de serre.

Tableau nr.1

Fig.	1	2	3	4	5	6	7
ηg	1,86	0,39	0,41	0,35	0,348	0,46	0,31
Conditions du travail	verre sale plaque sale	instalation propre ,	instalation propre	verre et feuille polyéstere	verre et feuille	$\alpha = 35^0$	$\alpha = 8^0$
Conditions météorologiques	ciel nuageux	ciel clair	ciel variable	ciel clair	ciel clair	ciel clair	ciel nuageux

Conclusions

L'emplecement des stations solaire pour le chauffage du bitume trop rapprochées des installations qui preparent la "mixture asphaltique" non est induque à cause de la poussière la quelle conduit à l'opacité et comme suite les températures réalisées et le rendement de station solaire s'abaissem si on ne prend pas de mésures pour le lavage souvent de la surface de l'installation, au moin une fois par semanine.

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CERCETARI EXPERIMENTALE ASUPRA INFLUENŢEI AGENTILOR EXTERIORI ASUPRA RANDAMENTULUI UNUI CAPTATOR CILINDRIC

Rezumat

In lucrare s-a studiat influenţa parametrilor meteorologici şi a poluării mediului asupra randamentului unui captator cilindric, utilizat pentru preîncălzirea şi menţinerea la temperatură constantă a bitumului din rezervoare de mare capacitate.

Din studiul efectuat s-a desprins faptul că, randamentul are o valoare maximă în cazul în care instalaţia este curată şi orientată pe direcţia sud.

Seminarul de Matematică şi Fizică
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Noiembrie 1983

A PROBABILITY DISTRIBUTION FUNCTION FOR THE ELECTRONIC STOPPING POWER

Dorobanţu V.T.

1. Introduction . In a previous paper [1] , it was already shown that the dissipative process, in which a recoilling nucleus, produced in a nuclear reaction and loosing its energy, mainly by interactions with the target electrons, can be treated according with the theory developed by Haken [2],[3],[4]. Following this general procedure, one can get a nonlinear form for the electronic stopping power

$$-\frac{dE}{dx} = k_1 v - k_3 v^3 \tag{1}$$

k_1, k_3 being positive constants, and v the speed of the nucleus.

The expression (1) is equivelent with the following one

$$m\frac{dv}{dt} = -k_1 v + k_3 v^3 \tag{2}$$

Writing $-k_1 v + k_3 v^3 = -\frac{dU}{dv}$, one obtains

$$U(v) = \frac{1}{2} k_1 v^2 - \frac{1}{4} k_3 v^4 \tag{3}$$

The term $-\frac{1}{4} k_3$ being negative, we have global instability [5] and because $\frac{1}{2} k_1 > 0$, we have a local stability. The state in which $\frac{1}{2} k_1 > 0$ is an "attractor".

The passing from one stable state to another stable state, is called in physics, the exchange of stabilities, and in mathematocs bifurcation.

The order parameter equation (2) can also incorporate the fluctuations, namely a force taking into account the impulses of equal magnitude felt by the nucleus, due to the target electrons, in foreward and backward directions. This random force may assume

the form

$$F_1(t) = \zeta \sum_j (-1)^{n_j} \delta(t-t_j) \tag{4}$$

where ζ stands for the size of the impulses, n_j is a random variable with values 0 or 1, and t_j a random time sequence.

With this force incorporated in equation (2) one gets

$$\frac{dv}{dt} = -\alpha_1 v + \alpha_2 v^3 + F(t) \tag{5}$$

where $\alpha_1 = \frac{k_1}{m}$, $\alpha_2 = \frac{k_3}{m}$, $F(t) = \frac{F_1(t)}{m}$ (6)

Because impulses in positive and negative directions are assumed of equal size, the statistical average of F(t) is

$$\langle F(t) \rangle = 0 \tag{7}$$

Let F(t) be

$$F(t) = F_o \sum_j (-1)^{n_j} \delta(t-t_j) \tag{8}$$

One may show that

$$\langle F(t)\, F(t') \rangle = \frac{F_o^2}{t_o} \delta(t-t') = \mathcal{C} \delta(t-t') \tag{9}$$

2. <u>The Fokker-Planck equation</u>. In treating the equation (2) it is avantageous to proceed to the Fokker-Planck equation

$$\frac{\partial f(v,t)}{\partial t} = -\frac{\partial}{\partial v}\left[A(v) f(v,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial v^2}\left[B(v)\, f(v,t)\right] \tag{10}$$

f(v,t) stands for the probability of finding the nucleus with the veolocity v in the interval dv at time t.

A(v) and B(v) being the drift vector and diffusion coefficient, respectively, which in our case take the forms

$$A(v) = -\alpha_1 v + \alpha_2 v^3 = -\frac{dU}{dv} \tag{11}$$

$$B(v) = \frac{F_o^2}{t_o} = \mathcal{C} \tag{12}$$

$\mathcal{C}$ being the correlation factor in (9).

The equation (10), now becomes

$$\frac{\partial f(v,t)}{\partial t} = -\frac{\partial}{\partial v}\left[\left(-\frac{dU}{dv} f\right) - \frac{1}{2}\mathcal{C}\frac{\partial f}{\partial v}\right] \tag{13}$$

The distribution function f(v,t) is of great importance, because it governs the stability, the fluctuations and the dynamics of the system [5] .

3. The time dependent case. To solve the equation (13) is a formidables task, but according to the Schwarts's theorem[6], a function f(x,y) may be expanded in an infinite serie

$$f(x,y) = \sum_j f_j(x)\, g_j(y). \qquad (14)$$

The question is : may we restrict ourselves at only one term in the serie of f(v,t) ? It seems yes, if we require f(v,t) be an analytic function in t. It can be the case here, because the times involved in the stopping process are. Of the order of 10^{-12} sec, and the Taylor's serie in t is rapidely converbent. As a convergent, we can write

$$f(v,t) = g(v).\, h(t) \qquad (15)$$

According to (15), the equation (13) decouples into two equations

$$\frac{dh(t)}{h(t)} = -\lambda\, dt\ ,\ h(t) = \ell_1\, e^{-\lambda t} \qquad (16)$$

and

$$\frac{d^2g}{dv^2} + \frac{2}{\ell}\frac{dU}{dv}\cdot\frac{dg}{dv} + \frac{2}{\ell}\left(\lambda + \frac{d^2U}{dv^2}\right) g = 0 \qquad (17)$$

In equation (17) one can look for a solution of the form

$$g(v) = g_1(v)\, \exp\left(-\frac{2U}{\ell}\right) \qquad (18)$$

where $g_1(v)$ is a solution of the following equation

$$\frac{d^2g_1}{dv^2} - \frac{2}{\ell}\frac{dU}{dv}\cdot\frac{dg_1}{dv} + \frac{2}{\ell}\lambda\, g_1 = 0 \qquad (19)$$

$g_1(v)$ takes the from

$$g_1(v) = \sum_{k=0}^{\infty} a_k v^k \qquad (20)$$

with

$$a_{k+4} = \frac{2}{\ell}\,\frac{\left[(k+2)\alpha_1 - \lambda\right] a_{k+2} - k\,\alpha_2 a_k}{(k+3)(k+4)} \qquad (21)$$

$k = 0,1,2,3,\ldots$

and

BUDAPESTI MŰSZAKI EGYETEM KÖNYVTÁRA 1848

$$a_2 = -\frac{\lambda}{\ell}\, a_0, \quad a_3 = \frac{2}{\ell}\,\frac{(\alpha_1-\lambda)a_1}{6} \tag{22}$$

$f(v,t)$, now becomes

$$f(v,t) = \ell_1 \exp(-\lambda t)\cdot \exp\left[-\frac{2U}{\ell}\right]\cdot\sum_{k=0}^{\infty} a_k v^k \tag{23}$$

4. Conclusions

One can see, that the stationary case can be obtained if $\lambda = 0$.

A good aproximation of $f(v,t)$ one might obtain, taking into account that, in fact v stands for $\frac{v}{c}$, with c - speed of light, when the terms with $k > 2$ cancel.

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FUNCTIA DISTRIBUTIEI DE PROBABILITATE PENTRU PUTEREA ELECTRONICA DE STOPARE

Rezumat

Plecînd de la ecuația Fokker-Planck, pentru $-\frac{dE}{dx}$ se tratează cazul dependent de timp, deducîndu-se astfel forma funcției de distribuție a probabilității $f(v,t)$.

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LE MODELAGE DU POTENTIEL DE MORSE PAR UN POTENTIEL HARMONIQUE DÉPLASÉ PAR L'USAGE DU FORMALISME DE LA MATRICE DE LA DENSITÉ

Popov D.

Le potentiel de Morse est un des potentielles qui reflète correctement les particularitées qualitatives des spectres de vibration des molécules diatomiques. Mais, quoique le problème des fonctions et des valeurs propres de l'équation de Schrödinger est complètement résolu [1] , le calcul des valeurs moyennes du déplacement des noyaux en utilizant ces fonctions est difficile et il ne peut pas faire exactement. Pour cette raison, dans les plusieurs cas, on préfère l'utilisation des fonctions de l'oscillateur harmonique au lieu des fonctions propres de l'oscillateur de Morse.

Malgré toutes les symplifications qu'elles les amène , l'utilisation des fonctions propres de l'oscillateur harmonique présente d'autres désavantages, par exemple, à cause de la parité des fonctions, les valeurs moyennes des puissances impaires du déplacement des noyaux ne peuvent être calculées. Dans cette situation on appèle à l'utilisation des diverses méthodes approximatives, par éxemple celles de tronquer le dévelopement en série de la fonction de Morse [2] ,[3] .

Dans cet ouvrage on propose de trouver les parametres : ω_0 - fréquence classique propre et a_0 - le déplacement moyen par rapport à la position d'équilibre à cause de l'anharmonicité d'un oscillateur harmonique caracterisé par l'hamiltonien :

$$\mathcal{H}_0 = \frac{p^2}{2m} + \frac{m\omega_0^2}{2}(x-a_0)^2, \qquad (1)$$

qui doit modeler le plus meilleurs le comportement d'oscillateur

anharmonique Morse, caracterisé par l'hamiltonien :

$$\mathcal{H} = \frac{p^2}{2m} + V(x) \equiv \frac{p^2}{2m} + D_e\left(1 - e^{-\alpha x}\right)^2. \qquad (2)$$

Ici, D_e este l'énergie de dissociation de la molécule, mésurée par rapport au niveaux minimum de la fonction potentielle de Morse, m - la masse réduite de la molécule et α - constante de Morse d'anharmonicite.

Pour calculer les valeurs moyennes on a utilisé la matrice de la densité dans répresentation de la coordonnée, qui est obtenue par la résolution d'équation de Bloch pour l'oscillateur harmonique (1) [4] :

$$\rho(x,x') = \frac{1}{(2\pi)^{\frac{1}{2}} l_0} e^{-\left[A(x-a_0)^2 + A(x'-a_0)^2 + B(x-a_0)(x'-a_0)\right]} \qquad (3)$$

où, on a noté :

$$l_0^2 \equiv \langle (x-a_0)^2 \rangle = \frac{\hbar}{2m\omega_0} \operatorname{cth}\left(\frac{\hbar\omega_0}{2KT}\right), \qquad (4)$$

$$A = \frac{1}{2} \frac{m\omega_0}{\hbar} \operatorname{cth}\left(\frac{\hbar\omega_0}{KT}\right), \qquad (5)$$

$$B = -\frac{m\omega_0}{\hbar} \frac{1}{\operatorname{sh}\left(\frac{\hbar\omega_0}{KT}\right)}. \qquad (6)$$

Pour obtenir les deux parametres cherchés , ω_0 et a_0 , on a utilisé la condition qu' a l'équilibre, dans le cas d'états stationaires, la force moyenne doit être nulle [4] :

$$\left\langle \frac{dV(x)}{dx} \right\rangle = 0, \qquad (7)$$

aussi que la théorème du viriel :

$$\left\langle \frac{p^2}{2m} \right\rangle = \frac{1}{2} \left\langle x \frac{dV(x)}{dx} \right\rangle. \qquad (8)$$

Cettes deux conditions permettent de trover les parametres cherchés seulement des données de la fonction potentielle (2).

Après avoir faire les calculs, la relation (7) conduit à :

$$a_0 = \frac{3}{2}\alpha l_0^2. \qquad (9)$$

On calcule la partie gauche de la rélation (8) d'apres la formule [4] :

$$\left\langle \frac{p^2}{2m} \right\rangle = -\frac{\hbar^2}{2m}\int_{-\infty}^{+\infty} \frac{\partial^2 \rho(x,x')}{\partial x^2}\bigg|_{x'=x} \cdot dx = \frac{m\omega_0^2}{2} l_0^2. \qquad (10)$$

où, après avoir faire la derivation, on remplace la variable x' avec x et, on intègre.

Finalement et tenant compte de la relation (9), on obtient l'equation transcendente :

$$D_e \alpha^2 e^{-\alpha^2 l_0^2} = \frac{m\omega_0^2}{2}, \qquad (11)$$

avec l'inconnue ω_0. On a résolu cette equation par la méthode graphique.

En analysant le graphique on peut constater que la solution ω_0 satisfait l'inegalité :

$$\omega_0 < \omega_e = \alpha\sqrt{\frac{2D_e}{m}}. \qquad (12)$$

Le déplacement moyen par rapport à la position d'équilibre à cause de l'anharmonicité, a_0, s'obtient ensuite, à l'aide des rélations (9) et (4) :

$$a_0 = \frac{3}{2}\alpha \frac{\hbar}{2m\omega_0} \cdot \operatorname{cth}\left(\frac{\hbar\omega_0}{2KT}\right). \qquad (13)$$

La méthode a était téstée pour quelques molécules diatomiques. Les résultats sont présentées dans le tableau ci-dessous.

Les constantes spectroscopiques sont prises du livre [5]. Tous les calculs ont été effectués pour T = 294°K , sauf la molécule O_2, pour laquelle T = 300°K .

Les resultats obtenues concordes aux ceux-là de l'ouvrage [3], où on fait les calculs pour les molécules de I_2 et de O_2 , pour les mêmes temperatures, en utilisant une fonction potentielle de trois termes.

Le principal avantage de notre méthode est celui qu'il permet, dans le cadre d'aproximation harmonique, le calcul de la valeur moyenne du déplacement x par rapport à la position d'equi-

libre des noyaux :

$$\langle x \rangle = \langle (x - a_0) + a_0 \rangle = a_0 . \qquad (14)$$

Tableau de résultats

Mol	Terme électronique	ω_e (cm^{-1})	ω_0 (cm^{-1})	a_0 (Å)
H_2	$X^1\Sigma_g^+$	4401	4340,023	0,0220031
HCl	$X^1\Sigma^+$	2991	2964,455	0,0152602
I_2	$X^1\Sigma_g^+ (0_g^+)$	215,115	214,475	0,0058713
HF	$X^1\Sigma^+$	4139	4093,719	0,0146
O_2	$X^3\Sigma_g^+$	1580	1573,964	0,0048008

Les valeurs obtenues pour a_0 sont bien en concordance aux celles de l'ouvrage [2] , où on a utilisé la méthode de tronquer la fonction potentielle de Morse, aussi que aux résultats experimenteaux obtenus par la méthode électronographique [3].

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MODELAREA POTENTIALULUI MORSE PRINTR-UN POTENTIAL ARMONIC DEPLASAT UTILIZIND FORMALISMUL MATRICII DENSITATII

Rezumat

In lucrare se deduce frecvenţa clasică proprie şi deplasarea medie faţă de poziţia de echilibru, datorită anarmonicităţii, ale unui potenţial armonic, cu care se fac calcule concrete pentru cîteva molecule biatomice. Rezultatele concordă bine cu datele experimentale.

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A p-n JUNCTION ELECTRONIC THERMOMETER

Erouţa A.

Electrical properties of semiconductor devices strongly depend on temperature and this in generally an undesind feature. The present work suggests a thermometric application of p-n diode based on its thermic sensitivity.

Theoretical equation of p-n diode characteristic is given [1] as

$$j = q n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) \left(e^{\frac{qU}{kT}} - 1 \right) + \frac{q n_i}{2\tau_o} \left[\frac{2\varepsilon}{q} \left(\frac{1}{N_D} + \frac{1}{N_A} \right) \left(U_{bo} - U \right) \right]^{1/2} \left(e^{\frac{qU}{2kT}} - 1 \right) \quad (1)$$

where $n_i = 2 \left(2\pi k T h^{-2} \right)^{3/2} \left(m_n^* m_p^* \right)^{3/4} e^{-\frac{E_g}{2kT}}$

is the intrinsec carrier concentration and $U_{bo} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$ the junction barrier height.

The first term in (1) is the diffusion current density white second one is due to the generation-recombination proces.

When direct polarity $U > 0$ is applied, usualy eq.(1) may be approached [1] at

$$j = j_o \left(e^{\frac{qU}{\eta kT}} - 1 \right) \quad (2)$$

where η is equal to 1 or 2 as diffusion or generation-recombination effect prevails, where

$$j_{o\,diff} \sim T^{\alpha} e^{-\frac{E_g}{kT}} \qquad j_{o\,gr} \sim T^{\beta} e^{-\frac{E_g}{2kT}} \quad (3)$$

α and β for silicon are 2,5 and 1,5.

It The applied voltage $U \sim 0{,}4 \div 0{,}7$ V, current density may be written

$$j = C T^{\alpha} e^{-\frac{E_g - qU}{kT}}$$

C = constant (4)

Taking into account that energy gap decreases stowly with temera-

ture

$$E_g = E_{g_o} - \gamma T \qquad \gamma \simeq 1{,}33.10^{-4}\ eV\ K^{-1}, \quad 2 \qquad (5)$$

and logarithming (4) one can write

$$U = U_o - a\theta \quad \text{with} \quad U_o = \frac{1}{q} E_{g_o} - aT_o\ , \quad a = \frac{1}{q}\left(k \ln C T^{\alpha} j^{-1} + \gamma\right), \quad \theta = T - T_o \qquad (6)$$

If j density is kept constant and $\Delta T < 200$ K, equation (6) becomes linear as figure 1 proves, where measurements data of 1N4148 (IPRS) silicon diode are given. Fig.2 shows an electronic

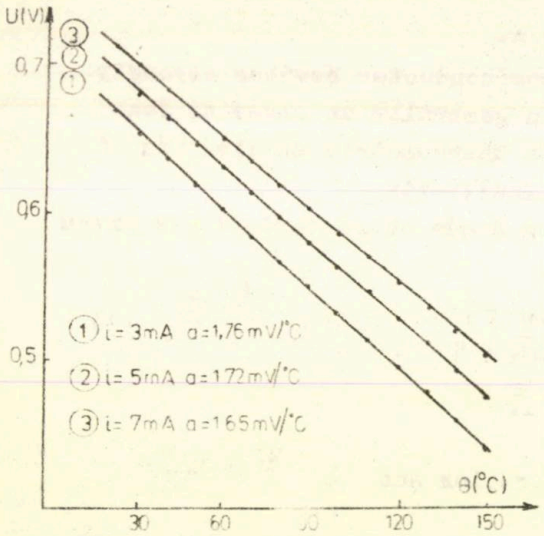

Fig.1 Temperature depending of the applied voltage

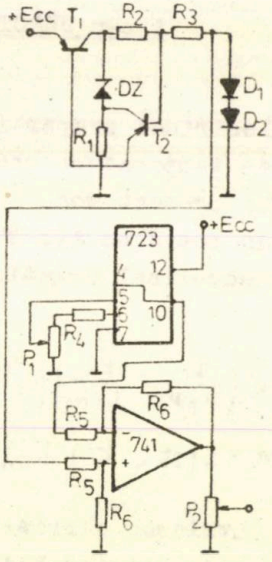

Fig.2 Electronic thermometer

thermometer designed on base of eq.(6) and fig.1 : the U voltage from D_1D_2 silicon diode pair through whrich current is held constant by T_1,T_2, D 2, R_2 current regulator, is applied on the inverting imput of a 741 differential amplifier, while the non inverting one receives a refference voltage U_r from the integrated voltage regulator 723C, choosing by P_1 pot. $U_r = 2U_o$, the output voltage will depend linearly on θ .

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TERMOMETRU ELECTRONIC CU JONCTIUNE p-n

Rezumat

In lucrare este prezentat un termometru electronic care furnizează la ieşire o tensiune care depinde liniar de temperatură.

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ELECTRONIC HYSTERESIS LOOP TRACER

Ercuţa A., Mihalca I.

The present paper describes an original electronic hysteresis loop tracer based on e.m. induction, with improved parameters thau otherones as given in [1] and [2] ; it was designed for chrystalline and amorphous ferromagnetic thin films, or swal quantities of substance.

Figure 1 shows the main electrical circuits of the device and its work maybe resumed as follows : by means of au AF generator, two sinusoidal magnetic fields H_1 and H_2 in B_1 and B_2 field wils cores are created, so that in the b_1 pick-up coil (surrounding a ferromagnetic sample p) and in b_2, b_3 compensation coils, e_i (i = 1,2,3) e.m.f. are induced

$$e_1 = \alpha_1(\dot{H}_1 + \mu_o^{-1}\dot{J}_{\parallel}) \quad e_{2,3} = \alpha_{2,3}\dot{H}_2 \tag{1}$$

with α_i constants depending on frecquency and parameters of b_i. Fraction η_1 and η_2 of G_1e_1 and e_2 are applied on A_2 inputs ; trimming phase and amplitude of e_2 up to close corresponding values of $G_1\eta_1\dot{H}_1$ signal (preliminary compensation), by means of $R\varphi_2$ and r_2P_2 divider the output is (G_K , K = 1,...,5 are the gains of A_K amplifiers)

$$u_{o2} = \eta_1 G_1 G_2 \alpha_1 \mu_o^{-1}\dot{J}_{\parallel} + \delta u_{2\,max}\sin(\omega t + \varphi_2) \tag{2}$$

To avoid δu_2 signal, differential A_4 amplifier is used by applying on its inputs u_{o2} and u_3 voltages, the last one given by driving e_3 through plase shifting Bridge (R_o, C_o, $R\varphi_3$), A_3 amplifier and r_3 P_3 divider

$$u_3 = 0{,}5\eta_3 G_3 e_{3\,max}\sin(\omega t + \varphi_3) \tag{3}$$

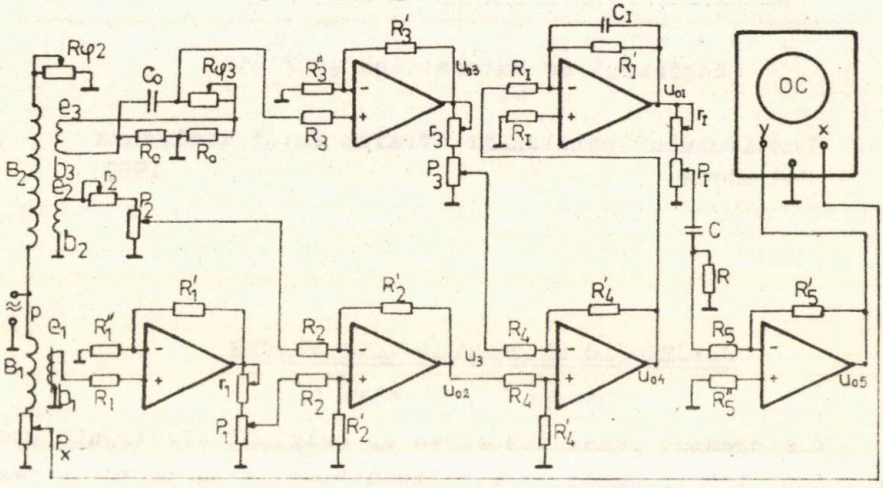

Fig.1 Electrical diagram of tracer

Choosing $u_{3\ max} = \delta\, u_{2\ max}$ and $\varphi_3 = \varphi_2$, the output is

$$u_{o4} = G_1 G_2 G_4 \alpha_1 \eta_1 \dot{J}_{11} \qquad (4)$$

Time integrating u_{o4} by means of RC integrator I gives

$$u_{oI} = -R_I^{-1} C_I^{-1} \int u_{o4}\, dt \qquad (5)$$

of which η_I fraction is applied on the inverting input of A_5, so that

$$u_{oI} = \beta J_{11}\ ,\ \beta = R_I^{-1} C_I^{-1} G_1 G_2 G_4 G_5 \alpha_1 \eta_1 \eta_I \ (V/T) \qquad (6)$$

The total integration error [3] $\delta = (\omega R_I' C_I)^{-1}$ is $\delta = 3{,}4.10^{-3}$ % if f = 100 Hz, $R_I' = 21{,}4$ MΩ, $C_I = 2{,}2$ μF. P_x potentiometer supplies a $u_x = kH_1$ voltage. Hence, applying u_{o5} and u_x on Y and X inputs of OC oscillograph, hysteresis loop J = f(H) is displayed on the screen. β and k constants may be found by calibration techniques.

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INSTALATIE PENTRU OBTINEREA CURBEI DE HISTEREZIS MAGNETIC

Rezumat

Lucrarea propune o instalaţie pentru ridicarea ciclului d de histerezis magentic ; eroarea totală de integrare este $\delta \sim 0{,}001$ % .